Porównanie dopasowania kilku liniowych efemeryd z modelem trzeciego ciała dla grupy gwiazd zaćmieniowych wykazujących duże zmiany O-C.

Damian Jabłeka

Obserwatorium Astronomiczne Uniwersytetu Jagiellońskiego

12 maja 2015



P - Period.

P = const if not of following occur:

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 - mass loss,

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 - mass loss,
 - mass exchange,
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 - pulsation.

If we know:

• period *P*,

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we can predict further minima:

If we know:

- period *P*,
- time of one particular minimum JD_0

we can predict further minima:

$$JD = JD_0 + EP$$
 $E = floor\left(rac{t-JD_0}{P}
ight) + 1$

i



Sometimes we miss minimum.



- C Calculation,
- O Observation.

To analyze difference we build O-C(E) diagrams

O-C types



Fittin (JD_0, P) - new exact ephemerides.

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What can cause sine-like shape?



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A₃ -semi amplitude, c - speed of light, a₁₂ - semi-maior axis, i - inclination.



A₃ -semi amplitude,
c - speed of light,
a₁₂ - semi-maior axis,
i - inclination.

$$A_3 \cdot c = a_{12} \sin i$$

if orbit is circular, but generally is not

$$a_{12}\sin i = \frac{A_3 \cdot c}{\sqrt{1 - e^2 \cos^2 \omega}}$$

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$$f(M_3) = \frac{(a_{12}\sin i)^3}{p_3^2}$$

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$$f(M_3) = \frac{(a_{12}\sin i)^3}{p_3^2} = \frac{(M_3\sin i)^3}{(M_1 + M_2 + M_3)^2}$$

٠

$$a_{12}\sin i = \frac{A_3 \cdot c}{\sqrt{1 - e^2 \cos^2 \omega}}$$

Knowing a_{12} and p_3 we can calculate mass function:

$$f(M_3) = \frac{(a_{12}\sin i)^3}{p_3^2} = \frac{(M_3\sin i)^3}{(M_1 + M_2 + M_3)^2} = \left[\frac{1}{p_3^2}\frac{A_3 \cdot c}{\sqrt{1 - e^2\cos^2\omega}}\right]^3$$

from this we can calculate lower mass limit $M_3 \sin i$

$$a_{12}\sin i = \frac{A_3 \cdot c}{\sqrt{1 - e^2 \cos^2 \omega}}$$

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from this we can calculate lower mass limit $M_3 \sin i$. Limit - because of inclination *i*.

O-C changes with orbit shape



Time of minimum delay:

$$\Delta \tau = \frac{A_3}{\sqrt{1 - e^2 \cos^2 \omega}} \cdot \left[\frac{1 - e^2}{1 + e \cos \nu} \sin(\nu + \omega) + e \sin \omega\right]$$

where u true anomaly, $u =
u(T_0, p_3).$

Time of minimum delay:

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0 - C =

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u(T_0, p_3).$

O - C = JD

Time of minimum delay:

$$\Delta \tau = \frac{A_3}{\sqrt{1 - e^2 \cos^2 \omega}} \cdot \left[\frac{1 - e^2}{1 + e \cos \nu} \sin(\nu + \omega) + e \sin \omega\right]$$

where u true anomaly, $u =
u(T_0, p_3).$

 $O - C = JD - JD_0$

Time of minimum delay:

$$\Delta \tau = \frac{A_3}{\sqrt{1 - e^2 \cos^2 \omega}} \cdot \left[\frac{1 - e^2}{1 + e \cos \nu} \sin(\nu + \omega) + e \sin \omega\right]$$

where u true anomaly, $u =
u(T_0, p_3).$

 $O - C = JD - JD_0 - P \cdot E$

Time of minimum delay:

$$\Delta \tau = \frac{A_3}{\sqrt{1 - e^2 \cos^2 \omega}} \cdot \left[\frac{1 - e^2}{1 + e \cos \nu} \sin(\nu + \omega) + e \sin \omega\right]$$

where ν true anomaly, $\nu = \nu(T_0, p_3)$.

 $O - C = JD - JD_0 - P \cdot E - q \cdot E^2$

Time of minimum delay:

$$\Delta \tau = \frac{A_3}{\sqrt{1 - e^2 \cos^2 \omega}} \cdot \left[\frac{1 - e^2}{1 + e \cos \nu} \sin(\nu + \omega) + e \sin \omega\right]$$

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where ν true anomaly, $\nu = \nu(T_0, p_3)$.

$$O - C = JD - JD_0 - P \cdot E - q \cdot E^2 - \Delta \tau$$

Problem is classical inverse problem:

$$\{JD_i, \sigma_i\}_{i=1...M} \rightarrow (JD_0, P, q, A, e, \omega, T_0, p_3)$$

we can solve it minimising:

$$\chi^2_{LITE} = \sum_{i=1}^{M} \left(\frac{(O-C)_i}{\sigma_i} \right)^2$$

Using Monte Carlo Procedure.

Selection

AN ATLAS OF O-C DIAGRAMS OF ECLIPSING BINARY STARS Jerzy M. Kreiner Chun-Hwey Kim, Il-Seong Nha



Out of 1140 O-C diagrams collected by Kreiner et al. we have chosen 79 binaries which exhibits possible cyclic orbital period variations.

Third body results - large sample

Preliminary results using Petr Zasche Matlab procedure.



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Third body results - selected sample

	system	P3	Α	e	$f(M_3)$	M ₃ sin <i>i</i>
		[years]	[days]		[M _☉]	[M _☉]
	AH Cep	70.52	0.07	0.53	0.31	7.52
	BO Gem	84.32	0.16	0.24	2.91	5.91
	CC Her	57.87	0.06	0.41	0.31	1.96
Based on	V Cas	65.99	0.04	0.27	0.11	1.3
10 C 10 C	RW Cap	75.39	0.13	0.11	2.07	5.15
preliminary results it	RW Per	85.14	0.11	0.27	1.13	3.57
turned out that 20	RW Tau	65.65	0.04	0.48	0.11	1.28
turned out that Zo	RX Gem	69.23	0.1	0.3	1.16	3.65
systems may have a	RZ Aur	96.3	0.07	0.09	0.23	1.59
systems may have a	SVV Cyg	78.99	0.13	0.22	1.81	4.74
third body whose	SV Opn	70.90 57.74	0.1	0.0	1.30	4.12
enna boay, whose	JZ Calli T I mi	1/25	0.09	0.7	2.4	3.87
the lower mass limit		260.3	0.15	0.00	0.95	3 31
	TU Her	64.21	0.08	0.37	0.73	2.3
exceeds that of a	TW Lac	75.84	0.07	0.48	0.33	1.93
	TX Ari	41.57	0.11	0.44	4.82	7.66
neutron star.	TY Peg	130.14	0.08	0.64	0.27	1.65
Mayes Manta Carla	TZ Eri	51.73	0.04	0.18	0.11	1.1
we use monte Cano	U Cep	131.39	0.28	0.12	6.31	13.67
fitting procedure	V442 Cas	92.17	0.13	0.59	2.21	4.63
ntting procedure.	V602 Aq	115.32	0.14	0.73	1.13	4.26
	V913 Oph	67.86	0.06	0.39	0.27	1.62
	XY Leo	18.93	0.03	0.18	0.26	1.46
	Y Cam	112.62	0.31	0.39	12.63	16.13
	Y Leo	90.27	0.05	0.11	0.1	1.06
	Y Psc	32.38	0.03	0.61	0.27	2.02
	Y Z Aq	45.4	0.2	0.52	20.36	24.92

Basic statistic









abc term

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LTTE vs. FLE







Damian Jabłeka

LTTE vs. FLE

Absolute parameters quality

system	ref.	Р	M_1	M_2	R_1	R_2
		[days]	$[M_{\odot}]$	$[M_{\odot}]$	$[R_{\odot}]$	$[R_{\odot}]$
AH Cep	[6]	1.774738	15.8	13.7	6.3	5.8
BO Gem	[15]	4.068664	2.15	0.36	2.00	3.4
CC Her	[15]	1.734016	2.40	0.60	2.20	2.3
IV Cas	[7]	0.998515	1.98	0.81	2.13	1.8
RW Cap	[15]	3.392395	2.05	0.92	2.00	4.15
RW Per	[15]	13.198733	2.40	0.38	3.10	5.6
RW Tau	[12]	2.768831	2.55	0.55	2.25	3
RX Gem	[5]	12.208525	2.40	0.43	2.4	7.3
RZ Aur	[15]	3.010631	2.00	0.59	2.00	3.25
SW Cyg	[11]	4.573110	2.50	0.50	2.60	4.3
SW Oph	[15]	2.446005	2.40	0.60	2.20	2.85
SZ Cam	[10]	2.698464	16.6	11.9	9.4	5.4
T Lmi	[4]	3.019909	6.1	0.5	2.6	3.5
TT And	[15]	2.765109	2.15	0.62	2.00	3.1
TU Her	[15]	2.267031	1.27	0.51	1.40	2.6
TW Lac	[15]	3.037481	2.15	0.56	2.00	3.2
TX Ari	n.a.	2.691303	1.00	1.00	1.00	1
TY Peg	[15]	3.092222	2.15	0.30	2.35	2.6
TZ Eri	[2]	2.606101	1.97	0.37	1.69	2.6
U Cep	[3]	2.492911	4.30	2.15	3.10	4.50
V442 Cas	[15]	3.592201	1.75	0.32	1.80	3
V602 Aql	[15]	3.012500	2.15	1.85	2.00	1.85
V913 Oph	[14]	1.917343	1.9	0.45	1.9	2.25
XY Leo	[17]	0.284103	0.82	0.50	0.85	0.68
Y Cam	[13]	3.305570	1.70	0.41	2.95	3.07
Y Leo	[16]	1.686088	2.29	0.74	1.90	2.47
Y Psc	[11]	3.765833	2.80	0.70	3.06	3.98
YZ Aql	[15]	4.672488	2.05	0.60	2.00	4.4

[15] M. A. Svechnikov and E. F. Kuznetsova., 1990. Orbital period - chromospherically activity coupling. Applegate 1992Gravitational potential: $\Phi = -\frac{GM}{r} + \frac{1}{2}GQ_{ik}\frac{x^ix^k}{r^5}$ Quadrupole tensor: $Q_{ik} = 3J_ik - \delta_{ik}TrJ$ Inertia tensor: $J_{ik} = \int dmx_i x_k = \int d^3 x \varrho(x) x_i x_k$



Period changes: Luminosity Variations: Magnetic field:

$$\begin{split} \frac{\Delta P}{P} &= -9 \left(\frac{R}{a}\right)^2 \frac{\Delta Q}{MR^2} \\ \Delta L_{RMS} &= \pi \frac{\Delta E}{P_{mod}} \\ B^2 &= 10 \frac{GM^2}{R^4} \left(\frac{a}{R}\right)^2 \frac{\Delta P}{P_{mod}} \end{split}$$

system	B[kG]	$\Delta L_{RMS}[L]$	syster
AH Cep	13.45	0.002	TU H
BO Gem	85.35	860.350	TW L
CC Her	24.83	2.152	ΤΧ Α
IV Cas	8.89	0.045	ΤΥ Ρ
RW Cap	3.58	0.060	TZ E
	71.34	385.482	U Cer
RW Per	87.88	2686.380	
RW Tau	33.31	3.881	V442
RX Gem	64.33	916.013	V602
RZ Aur	34.02	23.666	
SW Cyg	53.18	151.960	V913
SW Oph	35.64	13.043	XY L
SZ Cam	16.66	0.004	Y Car
	9.78	0.001	Y Lec
T Lmi	57.47	178.985	Y Pso
TT And	33.65	0.001	
	33.65	15.679	YZ A

system	B[kG]	$\Delta L_{RMS}[L]$
TU Her	63.55	466.465
TW Lac	46.81	55.943
TX Ari	304.24	252926.100
TY Peg	17.63	2.272
TZ Eri	60.95	163.624
U Cep	4.94	0.032
	30.22	2.062
V442 Cas	64.38	1062.630
V602 Aq	53.26	196.993
	48.99	100.926
V913 Oph	27.03	23.506
XY Leo	27.79	5.422
Y Cam	16.42	4.819
Y Leo	21.97	2.137
Y Psc	4.13	0.010
	36.37	0.005
YZ Aql	204.31	20074.710



Spectral type

According to Applegate (1992), Lanza (2006) spectral type of at least one component > F3

system	1	2	ref
Ah Cen	B05 Vn	 B0 5 Vn	2005 0 1 120 000 K
SZ Cam	09.51/	B5\/	1075 Ap% 2655 36 320B
	A2	A 5	SK
V Cas	A 21/		2010DASP 122 1211K
DW(T-	A JV		1082 A 1 272 206D
	DOV	KUIV	1983ADJ272200P
SW/ Cum	A0 A0-		1907FA3F99274D
SVV Cyg	A Ze	KU CEUU	1980A% 26AS39265M
	AU	GSIII	1979A% 26AS 36 273C
	A56 V	KULIII	aasabs199818ds/038ds/038.html
U Cep	B8V	G8 - V	1986Ap% 26SS 125 219K
XY Leo	K0	K0	2006PASA23154D
Y Cam	A9IV	K1IV	2010 MNRAS 408 2149R
Y Leo	A3	K0	1980 BVS.17861G
Y Psc	K3	K0	1980A% 26AS39265M
BO Gem	A 2	K3IV	SK
CC Her	A0	G6IV	SK
RW Cap	A3	G 3I V	SK
RW Per	A5IVE	G9IV	SK
RZ Aur	A3	K0IV	SK
SW Oph	A0	G7IV	SK
TT And	A2	G7IV	SK
TU Her	F5	MIIV	SK
TW Lac	A2	KOIV	SK
TY Peg	Δ2	G6IV	SK
V442 Cas	A7	K2IV	SK
YZ Agl	Δ3	KSIV	SK
V913 Onh	7	7	7
	, ,	, 7	7
	Damian	Jahtaka	ITTE ELE



What can changes binary period

Mass loss



Mass exchange



Following Tout and Hall (1991), the relationship between the rate of period change and the rates of mass and angular momentum loss of the binary can be written as

$$\frac{\dot{P}}{P} = -2\frac{\dot{M}}{M} - \frac{3(M_2 - M_1)}{M_1 M_2}\dot{M}_2 + 3K$$
$$K = \frac{2}{3}\left(\frac{R_{es}}{A}\right)^2 \frac{M}{M_1 M_2}\dot{M}$$

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Assuming that the sudden mass loss of a component of a binary is proportional to its mass $(\Delta M_2 = (M_2/M)\Delta M)$ and the escape radius is equal to the separation of the binary $(R_{es} \simeq A)$,

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Assuming that the sudden mass loss of a component of a binary is proportional to its mass $(\Delta M_2 = (M_2/M)\Delta M)$ and the escape radius is equal to the separation of the binary $(R_{es} \simeq A)$,

$$\frac{\Delta P_s}{P} = \left(\frac{-1}{M} + \frac{3(M_2 - M_1)}{M_1M} + \frac{2M}{M_1M_2}\right)\Delta M$$

FLE results

system	DM ₁	DM ₂	DM ₃	DM4	DM5
	10 ⁻⁷ M⊙				
RW Tau	-12.5	96.9	-34.7	-45.7	84.9
AhCep	184.0	-2100.0	1340.0		
CFTuc	-7.4	203.0	- 278.0		
VCas	7.1	-71.6	62.2		
SWCyg	10.8	-125.0	115.0		
TLmi	-2.8	32.7	26.5		
TWLac	-0.8	84.9	-124.0		
YCam	14.4	- 99.4	- 126.0		
YZAql	-38.4	264.0	- 317.0		
BOGem	5.9	-113.0			
CCHer	8.7	-81.5			
RWCap	-18.6	218.0			
RWPer	7.3	- 76.7			
RXGem	7.2	- 62.2			
RZAur	-6.7	71.8			
SWOph	-15.3	146.0			
SZCam	-129.0	1270.0			
T TAn d	-2.2	79.0			
TUHer	9.4	-111.0			
TXAri	6.2	-119.0			
TYPeg	- 3.3	37.2			
TZEri	3.3	-48.1			
UCep	120.0	-379.0			
V602Aq	8.7	-235.0			
V913Oph	9.2	-86.6			
YLeo	10.9	- 80.9			
V442Cas	7.7				

LTTE

- 6 free parameters
- clear theory
- complex theory
- non separated parameters

FLE

- 4-10 free parameters
- not fully explained
- simple fit
- fully separated parameters

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We can compare fit quality in terms of sum of squares of residua.

Y Cam FLE



Y Cam LTTE



RW Tau FLE



RW Tau LTTE



TZ Eri FLE



TZ Eri LTTE



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• In the case of 10 systems, the sum of squares of residuals of the third body solution is smaller than the combination of linear ephemeris fit, while in the remaining 17 cases a better solution was derived for the combination of linear ephemeris.

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- In FLE idea more problematic is that we need mechanism of accretion matter onto the system from surrounding space.
- Fit quality can also be caused by very uneven data coverage, which can favor linear fit rather than sine-like one.

Confirmation

We use Robo-AO, the first robotic laser adaptive system at the Palomar 60-inch telescope to direct imaging of additional components in binary systems.

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All pictures cover 2.32×1.76 arcsec, in case of RW Per we measure the peak to peak distance to be equal to 172 mas

	BO Gem	RW Per	SW Cyg
•	1	•	•
	TT And	V442 Cas	V602 Aql
•		•	•

Confirmation

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The RW Per parallax from the Hipparcos catalogue equals to 3.28mas gives us distance from binary to third component $(52.39 \pm 3.14)au$. On the other hand, we can calculate the distance from the third body fitting. The O-C amplitude equal to 0.11days corresponds to the distance of $(19.52 \pm 5.68)au$.

Dynamic three body simulations



Dziękuję za uwagę!

jableka@oa.uj.edu.pl