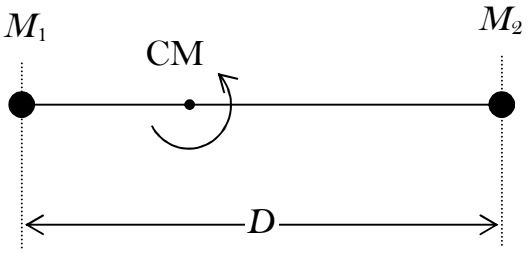
QUESTION 3 BINARY SYSTEM (10 points)



A binary star system consists of M_1 and M_2 separated by a distance D. M_1 and M_2 are revolving with an angular velocity w in circular orbits about their common centre of mass. Mass is continuously being transferred from one star to the other. This transfer of mass causes their orbital period and their separation to change slowly with time.

In order to simplify the analysis, we will assume that the stars are like point particles and that the effects of the rotation about their own axes are negligible.

a) What is the total angular momentum and kinetic energy of the system? (2 points)

b) Find the relation between the angular velocity w and the distance D between the stars. (2 points)

c) In a time duration Δt, a mass transfer between the two stars results in a change of mass ΔM_1 in star M_1, find the quantity Δw in terms of w, M_1, M_2 and ΔM_1. (3 points)

d) In a certain binary system, $M_1 = 2.9\ M_\odot$, $M_2 = 1.4\ M_\odot$ and the orbital period, $T = 2.49$ days. After 100 years, the period T has increased by 20 s. Find the value of $\frac{\Delta M_1}{M_1 \Delta t}$ (in the unit "per year"). (1.5 points)

e) In which direction is mass flowing, from M_1 to M_2, or M_2 to M_1? (0.5 point)

f) Find also the value of $\frac{\Delta D}{D \Delta t}$ (in the unit "per year"). (1 point)

You may use these approximations:

$(1+x)^n$: $1 + nx$, when $x = 1$;

$(1+x)(1+y)$: $1 + x + y$, when $x = 1, y = 1$.