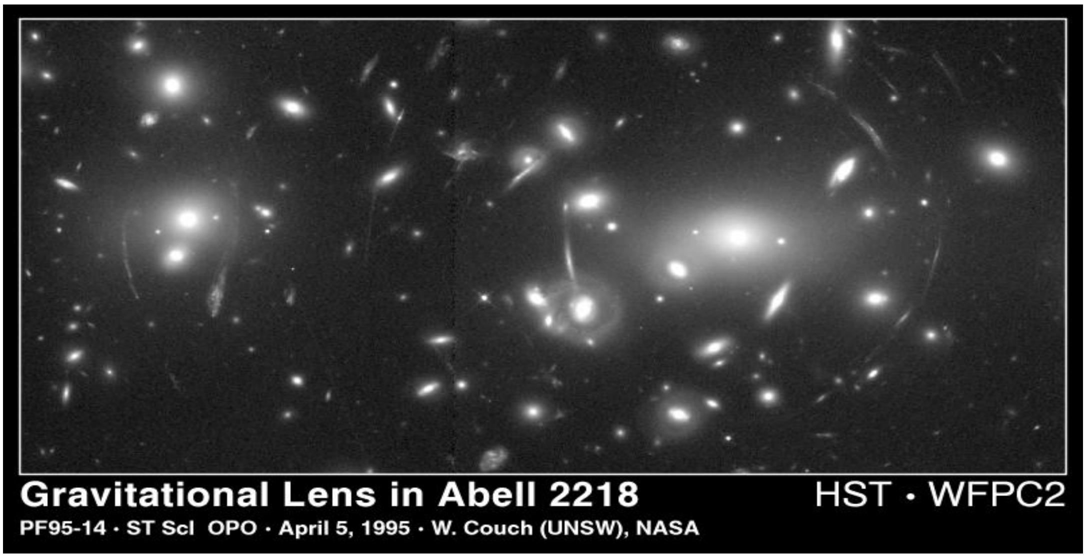

QUESTION 4 GRAVITATIONAL LENSING (10 points)



The deflection of light by a gravitational field was first predicted by Einstein in 1912 a few years before the publication of the General Relativity in 1916.
A massive object that causes a light deflection behaves like a classical lens.
This prediction was confirmed by Sir Arthur Stanley Eddington in 1919.

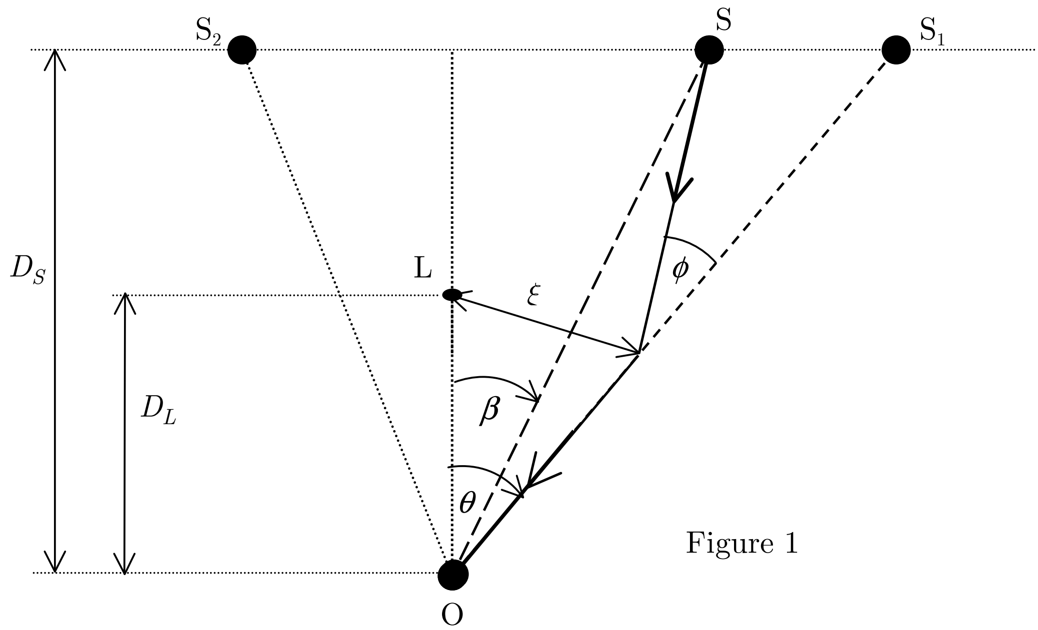


Figure 1

Consider a spherically symmetric lens, with a mass M with an impact parameter ξ from the centre. The deflection equation in this case is given by:

$$\phi = \frac{4GM}{\xi c^2} \quad \text{, a very small angle}$$

In figure 1, the massive object which behaves like a lens is at L. Light rays emitted from the source S being deflected by the lens are observed by observer O as images S_1 and S_2. Here, ϕ, β, and θ are very very small angles.

a) For a special case in which the source is perfectly aligned with the lens such that $\beta = 0$, show that a ring-like image will occur with the angular radius, called Einstein radius θ_E, given by:

$$\theta_E = \sqrt{\left(\frac{4GM}{c^2}\right)\left(\frac{D_S - D_L}{D_L D_S}\right)} \qquad \text{(2 points)}$$

b) The distance (from Earth) to a source star is about 50 kpc. A solar-mass lens is about 10 kpc from the star. Calculate the angular radius of the Einstein ring formed by this solar-mass lens with the perfect alignment. (1 point)

c). What is the resolution of the Hubble space telescope with 2.4 m diameter mirror? Could the Hubble telescope resolve the Einstein ring in b)? (2 points)

d). In figure 1, for an isolated point source S, there will be two images $(S_1$ and $S_2)$ formed by the gravitational lens. Find the positions (θ_1 and θ_2) of the two images. Answer in terms of β and θ_E. (2 points) e). Find the ratio $\frac{\theta_{1,2}}{\beta}$ ($\frac{\theta_1}{\beta}$ or $\frac{\theta_2}{\beta}$) in terms of η. Here $\theta_{1,2}$ represents each of the image positions in d.) and η stands for the ratio $\frac{\beta}{\theta_E}$. (2 points)

f). Find also the values of magnifications $\frac{\Delta\theta}{\Delta\beta}$ in terms of η for $\theta = \theta_{1,2}$ ($\theta = \theta_1$ or $\theta = \theta_2$), when $\Delta\beta \ll \beta$, and $\Delta\theta \ll \theta$. (1 point)