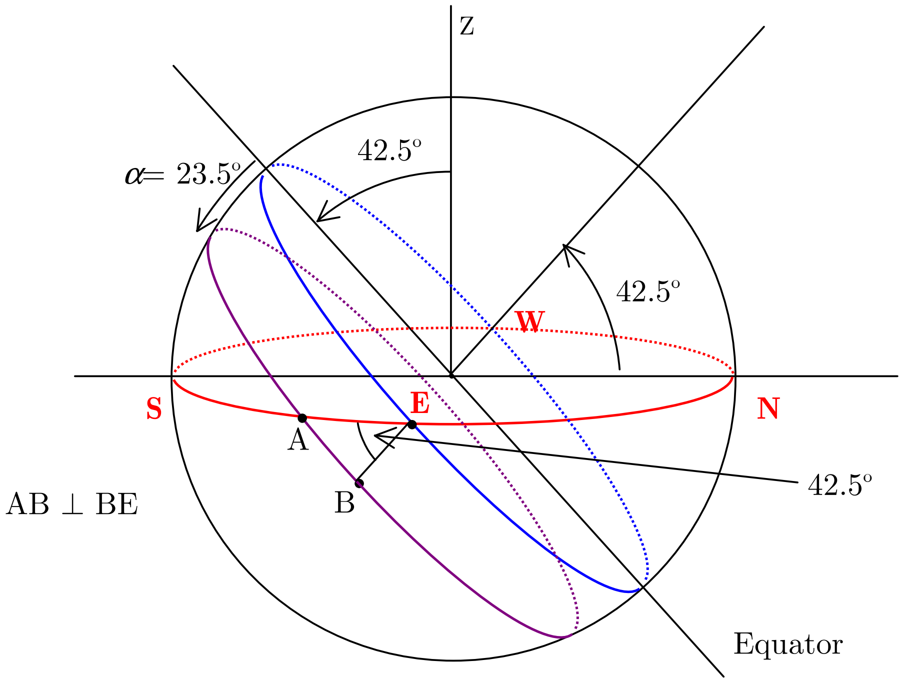

<u>SOLUTION FOR QUESTION 1.</u> (30 points for 15 short questions)

1.1



$$\text{E} = \text{East point}$$

$$\text{BE} = 23.5^\circ = \left|\text{declination of the Sun}\right|$$

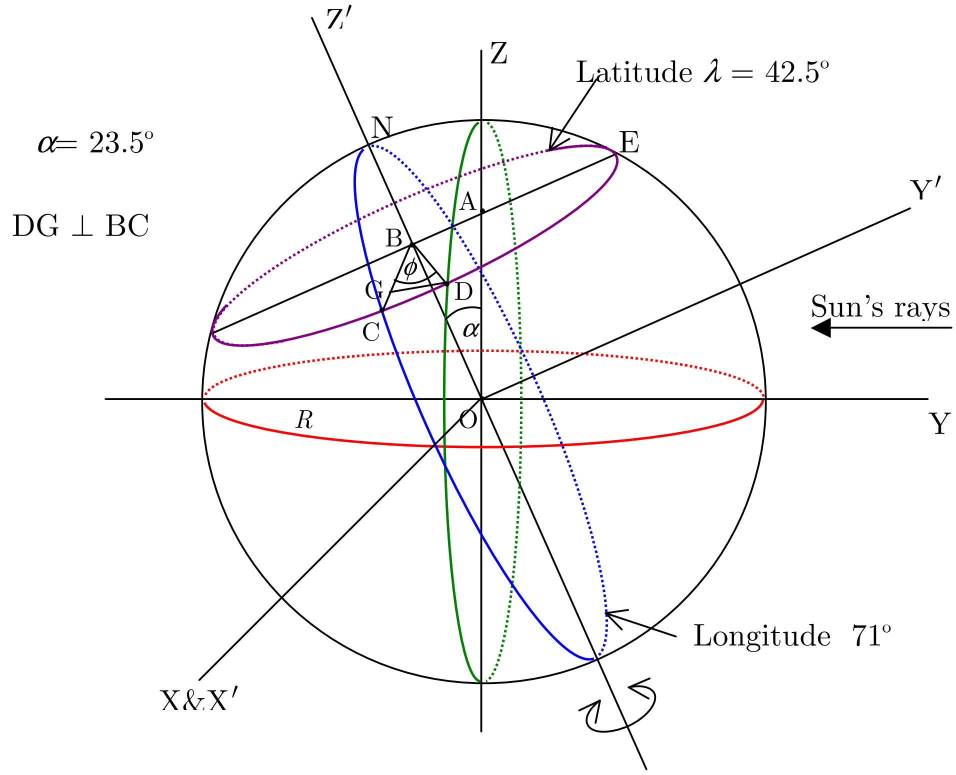
$$\frac{\text{AB}}{\sin 42.5^\circ} = \frac{\text{BE}}{\sin\left(90^\circ - 42.5^\circ\right)}$$

$$\text{AB} = \text{BE}\frac{\sin 42.5^\circ}{\cos 42.5^\circ} = 23.5^\circ \tan 42.5^\circ$$

local time = (23.5 tan 42.5° / 15) hrs after 6:00 = 7:26 am. The official time at 75° W should be 16 min. less.

Ans. 7:10 am.

Alternative solution (for 1.1)



The Earth's position relative to the Sun is shown in the figure.

Note that

$$\text{OB} = R\sin\lambda$$

$$\text{AB} = \text{OB}\tan\alpha$$

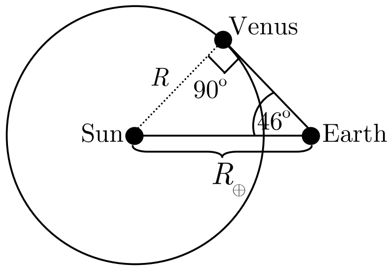
$$\text{BC} = \text{BD} = \text{BE} = R\cos\lambda$$

$$\sin\phi = \frac{\text{DG}}{\text{BD}} = \frac{\text{BA}}{\text{BD}} = \frac{\text{BO}\tan\alpha}{R\cos\lambda} = \tan\alpha\tan\lambda$$

$$= \tan\left(23.5^0\right)\tan\left(42.5^o\right) = 0.39843 = \sin(23.48^o)$$

Hence, the Sun will rise at $\left(71^o + 23.48^o\right)\times 4\,\text{min}. - 5$ hours. = 77.92 min. after 6 a.m.. This is at 7:18 a.m.

1.2



The angular separation is maximum when Sun, Venus and Earth form a right-angled triangle as shown.

Here $R = R_{\oplus} \sin 46^{\circ}$

$$= (1\,\text{A.U.}) \sin 46^{\circ}$$

$$= 0.72 \ \text{A.U.}$$

1.3 If the same face of the Earth were to face the Sun all the time then the Earth would make <u>one</u> complete turn relative to fixed stars in one solar year (365.25 solar days).

This implies that in 365.25 solar days our actual Earth makes (365.25+1) complete turns relative to fixed stars.

Hence 365.25 solar days are the same time interval as 366.25 sidereal days;

and $$183 \text{ solar days} \equiv \frac{183 \times 366.25}{365.25} \text{ sidereal days}$$

$$= 183.50 \quad \text{sidereal days}$$

<u>OR</u>

1 solar day = 24 x 60 x 60 s = 86400 s

1 sidereal day = 23 x 3600 + 56 x 60 + 4.1 = 86164.1 s

183 solar days = 183.50 sidereal days

1.4 During a full Moon we see the <u>whole face</u> of the Moon.

Hence $$\frac{\text{Moon's diameter}}{\text{distance to Moon}} = \text{angle in radians}$$

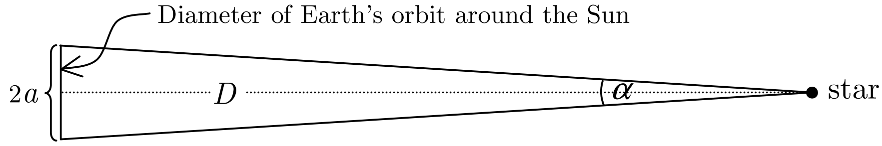
$$= \frac{0.46 \times \pi}{180}$$

$$\text{Distance to the Moon} = (\text{Moon's diameter}) \times \frac{180}{0.46 \times \pi}$$

$$= (2 \times 1.7374 \times 10^{6} \ \text{m}) \times \frac{180}{0.46 \times \pi}$$

$$= 4.328 \times 10^{8} \ \text{m} = 4.3 \times 10^{5} \ \text{km}$$

1.5



$$a = 1 \text{ A.U.} = 1.496\times10^{11} \text{ m}$$

$$1 \text{ pc} = 3.0856\times10^{16} \text{ m}$$

$$D = 100\times3.0856\times10^{16} \text{ m}$$

$$\alpha = \frac{2a}{D} = \frac{2\times1.496\times10^{11}}{100\times3.0856\times10^{16}} \text{ radian}$$

$$= 0.96966.....\times10^{-7} \text{ radian}$$

$$= 5.555779...\times10^{-6} \text{ degree}$$

$$= 0.02 \text{ arc second}$$

1.6 According to Kepler's third law we have

$$(\text{period})^2 = (\text{constant})(\text{semi-major axis})^3$$

$$T^2 = (\text{constant})a^3$$

This constant is $1\frac{(\text{year})^2}{(\text{A.U.})^3}$ when T is measured in years and a in A.U.'s.

For this comet we have

$$a = \frac{31.5+0.5}{2} = 16.0 \text{ A.U.}$$

$$T^2 = (16)^3 = 16\times4\times4\times16 = (64)^2$$

$$T = 64 \text{ years}$$

1.7 According to Kepler's second law we have;

The area is swept out at constant rate throughout the orbital motion.

Now, for this comet the area swept out in one orbital period is πab where a is the semi-major axis and b the semi-minor axis of the orbit.

$$a = 16.0 \text{ A.U.}$$

$$b^2 = a^2 - (\text{distance from a focus to centre of ellipse})^2$$

$$= (16.0)^2 - (16.0 - 0.5)^2$$
$$b = 3.968 \text{ A.U.}$$
$$\pi ab = 199.5 \ (\text{A.U.})^2$$

Hence, the area swept out per year is

$$\frac{199.5\,(\text{A.U.})^2}{64\,\text{years}} = 3.1 \ (\text{A.U.})^2 \ / \ \text{year}$$

1.8 From the Wien's displacement law

$$\lambda_{\max} T = 2.8977 \times 10^{-3} \ \text{m.K}$$
$$\lambda_{\max} = \frac{2.8977 \times 10^{-3}}{4000} \ \text{m} = 7.244 \times 10^{-7} \ \text{m}$$
$$= 724 \ \text{nanometers}$$

1.9 From the Stefan-Boltzmann law, the total power emitted is

$$L = (4\pi R^2)(\sigma T^4) = (4\pi R_\odot^2)(\sigma T_\odot^4)\left(\frac{R}{R_\odot}\right)\left(\frac{T}{T_\odot}\right)^4 = L_\odot (2.5)^2 \left(\frac{7500}{5800}\right)^4$$
$$\frac{L}{L_\odot} = (6.25)(2.796) = 17.47 = 17.5$$
$$L = 17.5 L_\odot$$

1.10 $$\text{flux density} = \frac{\text{luminosity}}{4\pi \times (\text{distance})^2}$$

$$\text{distance} = \sqrt{\frac{0.4 \times 3.826 \times 10^{26}}{4\pi \times 6.23 \times 10^{-14}}} = 1.398 \times 10^{19} \ \text{m}$$
$$= \frac{1.398 \times 10^{19}}{3.0856 \times 10^{16}} \ \text{pc} = 453 \ \text{pc}$$

1.11 $$L_{\text{supernova}} = 10^{10} L_{\odot}$$

Let D be the distance to that supernova.

Then the flux intensity on Earth would be $I = \dfrac{10^{10} L_{\odot}}{4\pi D^2}$

This must be the same as $\dfrac{L_{\odot}}{4\pi \left(D_{\oplus-\odot}\right)^2}$

$$\frac{10^{10} L_{\odot}}{4\pi D^2} = \frac{L_{\odot}}{4\pi \left(1 \text{ A.U.}\right)^2}$$

$$\therefore \quad D = 10^{\frac{10}{2}} \text{ A.U.} = 10^5 \text{ A.U.}$$

$$= \frac{1.496 \times 10^{16}}{3.0856 \times 10^{16}} \text{ pc} = 0.485 \text{ pc}$$

$$= 0.485 \times 3.2615 \text{ ly} = 1.58 \text{ ly}$$

1.12 The difference in frequencies is due to the relativistic Doppler shift. Since the observed frequency of emission from the gas cloud is higher than the laboratory frequency ν_0, the gas cloud must be approaching the observer.

Hence, $$\nu = \nu_0 \sqrt{\frac{c+v}{c-v}}$$

$$\frac{v}{c} = \frac{\left(\dfrac{\nu}{\nu_0}\right)^2 - 1}{\left(\dfrac{\nu}{\nu_0}\right)^2 + 1}$$

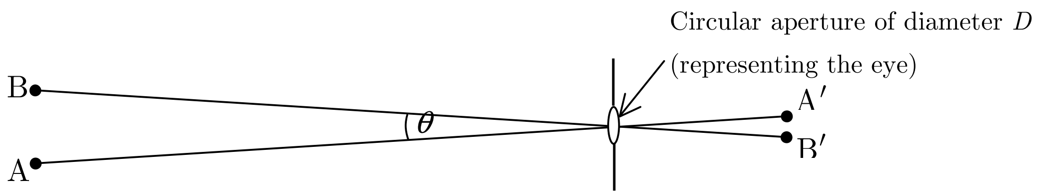
$$= \frac{0.001752379}{2.001752379} = 0.000875422$$

$$v = \left(0.000875422\right)\left(2.99792458 \times 10^8 \text{ m.s}^{-1}\right)$$

$$= 0.00262445 \times 10^8 \text{ m.s}^{-1}$$

$$= 262.445 \text{ km.s}^{-1}$$

1.13



A and B are two distant objects.

A′ and B′ are the central maxima of their diffracted images.

The angular position ϕ_1 of the first minimum relative to central maximum of each diffraction pattern is given by $\phi_1 = 1.22\dfrac{\lambda}{D}$.

According to Lord Rayleigh the minimum angle of resolution is ϕ_1.

Hence the images of A and B will be resolved if $\theta > \phi_1$, $D > 1.22\dfrac{\lambda}{\theta}$.

We may take distance AB to be the diameter of the crater. Hence the diameter D of the eye's aperture must be, at least, $D_{\min} = 1.22\dfrac{\lambda}{\theta}$.

$$\theta = \frac{80 \text{ km}}{\text{distance from Earth to Moon}}$$

$$= \frac{80\times 10^{3} \text{ m}}{3.844\times 10^{8} \text{ m}} = 2.081\times 10^{-4} \text{ radian}$$

$$\lambda = 500\times 10^{-9} \text{ m}$$

$$\therefore \quad D_{\min} = \frac{1.22\times 500\times 10^{-9}}{2.081\times 10^{-4}} \text{ m} = 2.93\times 10^{-3} \text{ m}$$

$$= 2.9 \text{ mm}$$

Hence, it is possible to resolve the 80 km-diameter crater with naked eye.

1.14 The spherical boundary at which the escape velocity becomes equal to the speed of light is of radius $R = \dfrac{2GM_{\odot}}{c^2}$

$$R = \frac{2\times 6.672\times 10^{-11}\times 1.989\times 10^{30}}{\left(2.99792458\times 10^{8}\right)^2} \text{ m}$$

$$= \frac{2 \times 6.672 \times 1.989}{2.998 \times 2.998} \times 10^3 \quad \text{m}$$
$$= 2.95 \quad \text{km}$$

1.15 The flux ratio versus magnitude difference implies

$$m_1 - m_2 = -2.5 \log\left(\frac{f_1}{f_2}\right)$$
$$\frac{f_1}{f_2} = 10^{\frac{(m_2 - m_1)}{2.5}}$$

So for a magnitude difference of $(-1.5) - 6 = -7.5$ we find a flux ratio of

$$\frac{f_{\min}}{f_{\max}} = 10^{\frac{-7.5}{2.5}} = 10^{-3}$$

And thus the visible stars range only over a factor of 1000 in brightness.