
QUESTION 2 **A PLANET & ITS SURFACE TEMPERATURE**

SOLUTION

a.) Intensity $I = \dfrac{L}{4\pi D^2}$ (1 point)

b.) Absorption rate $\mathcal{A} = (1-\alpha)\pi R^2 I$

$$= (1-\alpha)\frac{LR^2}{4D^2}$$ (1 point)

c.) Light energy reflected by the planet per unit time is $\alpha\pi R^2 I = \dfrac{\alpha L R^2}{4D^2}$

(1 point)

Hence the planet's luminosity is

$$L_{\text{planet}} = \frac{\alpha L R^2}{4D^2}$$ (1 point)

d.) Here, we will neglect the planet's internal source of energy.

Let T be the black-body temperature of the planet's surface in kelvins.

Since the planet is rotating fast, we may assume that its surface is being heated up uniformly to approximately the same temperature T.

The total amount of black-body radiation emitted by the planet's surface is from Stefan-Boltzmann law: $4\pi R^2 \cdot \sigma T^4, \sigma$ being Stefan-Boltzmann constant.

At equilibrium, that is when the temperature remains steady, this emission rate must be equal to the absorption rate in b.). (1 point)

Hence

$$4\pi R^2 \cdot \sigma T^4 = (1-\alpha)\frac{LR^2}{4D^2}$$

$$T = \left[(1-\alpha)\frac{L}{16\pi\sigma D^2}\right]^{\frac{1}{4}}$$ (1 point)

e.) In this case the emitted black-body radiation is mostly from the planet's surface facing the star. The emitting surface area is now only $2\pi R^2$ and not $4\pi R^2$. Hence the surface temperature is given by T', where

$$2\pi R^2 \cdot \sigma (T')^4 = (1-\alpha)\frac{LR^2}{4D^2}$$ (1 point)

$$T' = \left[\left(1-\alpha\right)\frac{L}{8\pi\sigma D^2}\right]^{\frac{1}{4}} = \left(2\right)^{\frac{1}{4}} \cdot T \approx \left(1.19\right)T$$ (1 point)

f.)

$$T = \left[\left(1-\alpha\right)\frac{L}{16\pi\sigma D^2}\right]^{\frac{1}{4}}$$

$$T = \left[\left(1-0.25\right)\times\frac{3.826\times10^{26}}{16\pi\times5.67\times10^{-8}\times\left(1.523\times1.496\times10^{11}\right)^2}\right]^{\frac{1}{4}}$$

$$= 209.8 \simeq 210 \text{ K} = \text{-63°C}$$ (2 points)