Long theoretical questions – Question 4 (reserve) solution & points

Max 30 points

Stating that the body radiates losing thermal energy **3**

Determining the thermal energy of the body

The specific heat capacity of an ideal gas is **4**

$c_V = 12{,}5 J\cdot K^{-1}\cdot mol^{-1}$

or

$c_p = 21\, J\cdot K^{-1}\cdot mol^{-1}$

Both are reasonable as this is an estimate.

10 Jupiter masses is $2\cdot 10^{28}\, kg = 10^{31}\, mol$ **4**

Amount of energy corresponding to a change of ΔT will be

$\Delta E = m\cdot c\cdot \Delta T$ **4**

Calculation of energy lost through radiation

From the mass and density we calculate radius

$R = \sqrt[3]{\frac{3\cdot m}{4\pi\cdot \varrho}}$ = $1{,}69\cdot 10^{8}$ m. **3**

Radiated power is thus

$\Delta M = 4\pi\cdot R^2\cdot \sigma\cdot T^4$ **3**

and therefore energy

$\Delta E = 4\pi\cdot R^2\cdot \sigma\cdot T^4\cdot \Delta t$ **3**

comparing thermal energy and energy loss rate to get Δt

$$\Delta t = \frac{(m\cdot c\cdot \Delta T)}{4\pi\cdot R^2\cdot \sigma\cdot T^4}$$ **3**

The rate of cooling can be therefore calculated as

$$\frac{\Delta T}{\Delta t} = \frac{4\pi R^2 \sigma T^4}{mc}$$ **1**

Subsitituting we get about $3\cdot 10^4$ years / K, (so quite quickly). 2