Long theoretical questions – solutions and points

Question 1 – Transit of an exoplanet

Max 30 points

Calculation of the angular distance on the orbit which the planet travels during the transit **5**
Since the orbit is circular,

$$\sin\left(\frac{\alpha}{2}\right)=\frac{\pi t}{T}$$

where t and T are, respectively, the duration of the transit and the orbital period of the planet. We can leave the angle as $\sin\left(\frac{\alpha}{2}\right)$ as this will be useful later; the numerical value of this expression is about 0.112.

Calculation of the velocity of the planet **7**
The velocity v at the beginning and end of the transit and the measured difference in velocities Δv form an isosceles triangle, with the angle between the equal sides = 2α. Thus

$$2v\cdot\sin\left(\frac{\alpha}{2}\right)=\Delta v$$

from which we get a velocity equal to about 134 km/h. **2**

Calculation of the radius of the orbit **2**
since we know the velocity on a circular orbit, this is trivial:

$$2\pi r=vT\Rightarrow r=\frac{vT}{2\pi}$$

Substituting we get about $6.4\cdot10^{6}$ km. **2**

Calculation of the radius of the star
With the velocity on the orbit and the transit duration, we get the radius of the star:

$2R=tv$ **2**

$\Rightarrow R=\frac{tv}{2}$ = $7.5\cdot10^{5}$ km . The star is thus similar to the Sun in size. **3**

Calculation of the mass of the star :

Given the orbital speed and radius, do this in the usual way:

$$\frac{v^2}{r}=\frac{GM}{r^2}$$ **5**

thus

$$M=\frac{v^2 r}{G}$$

The mass is therefore about $1.8\cdot10^{30}$ kg, so again similar to the mass of the Sun. **2**

Question 3 – Proper motion

Max 30 points

Difference in radial distance is about $1{,}58 \cdot 10^{15}\,m$ 2
Difference in right ascension is about 2.4767^o 2
Difference in declination is about 1.84^o 2

Thus the angular distance on the sky is about 3.09^o 3

Calculating the linear distance perpendicular to the line of sight
Using trigonometric functions and the distance from the Sun, we obtain a distance of $2{,}1 \cdot 10^{15}\,m$
2

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The distance between the stars is thus $2{,}63 \cdot 10^{15}\,m$ 3

Calculate the difference in angulare velocity in R.A. and Dec. Calculations (same method) for R.A. and Dec give a difference of 0.18 arcsec/year in both. 2+2

Conversion of angular velocity on the sky to transverse linear velocity. (works out aroud 15) 4

Note that transverse velocity is a lower limit to total velocity 2.

Compare obtained velocity to velocity expected for the distances between stars.
At distances of tens of thousands of AU the velocity is too great for the stars to form a bound system even with very massive stars. Even for a pair of one light and one heavy star, the heavy one would have a mass of thousands of Solar masses. 4

Final answer and conclusion: 2