Theory, long N°2 -Brightness of an elliptical galaxy - standard solution

1)

The luminosity distance is defined to satisfy a canonical inverse square law of the bolometric luminosity L (total emitted radiation power) and observed flux F (flux-luminosity relationship)

$$F = \frac{L}{4\pi d_L^2} \tag{1}$$

Units of the flux and luminosity are expressed in SI ($W \cdot m^{-2}$ and W respectively), or CGS ($erg \cdot s^{-1} \cdot cm^{-2}$, $erg \cdot s^{-1}$ respectively).

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2)

Only the part of the spectrum (namely B-band) is observed, but the inverse square law is applicable also to the monochromatic luminosity $L_\lambda(\lambda)$ and monochromatic flux $S(\lambda)$ [1], taking into account that observed wavelengths are not equal to emitted ones. (Observers frame is not the same as a frame of galaxy). For a small wavelength interval

$$S(\lambda_o) \cdot \Delta\lambda_o = \frac{L_\lambda(\lambda_e)}{4\pi d_L^2} \cdot \Delta\lambda_e ,$$

Observed fraction of the total spectrum is redshifted into the frame of the observer, and a relationship between the observed and emitted wavelengths, λ_o and λ_e, can be expressed as

$$\lambda_e = \lambda_o / (1+z) .$$

So,

$$\boxed{S(\lambda)\Delta\lambda = \frac{1}{4\pi d_L^2} \cdot L_\lambda\left(\frac{\lambda}{1+z}\right) \cdot \frac{1}{1+z} \Delta\lambda .} \tag{2}$$

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3)

The absolute luminosity is defined as the flux would be observed at a distance of 10 pc from the compact source at rest (not redshifted). It means that the frame would be the same as for the object.

$$S_{10}(\lambda) = \frac{L_\lambda(\lambda)}{4\pi d_{10}^2} , \tag{3}$$

where d_{10} is the distance equal to 10 pc, $S_{10}(\lambda)$ denotes a monochromatic flux in the wavelength of λ measured at 10 pc from the object.

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[1] *Monochromatic flux is sometimes called as spectral density of flux (energy per unit time per unit area per unit wavelength) while monochromatic luminosity as spectral density of the luminosity (energy per unit time per unit wavelength). Their form is described by SED and thus spectral index.*

4)
Dividing eq. (2) by (3) we get

$$\frac{S(\lambda)}{S_{10}(\lambda)} = \left(\frac{d_{10}}{d_L}\right)^2 \cdot \frac{L_\lambda\left(\frac{\lambda}{1+z}\right)}{L_\lambda(\lambda)} \cdot \frac{1}{1+z} \qquad (4)$$

for all the wavelengths.
The lower edge of validity of SED approximation is equal to 250nm. This corresponds to 1.5·250 nm=375 nm what is below the blue cut-off of the B filter[2], so SED approximation is applicable. Using the SED of the galaxy, we get

$$\frac{L_\lambda\left(\frac{\lambda}{1+z}\right)}{L_\lambda(\lambda)} \cdot \frac{1}{1+z} = \left(\frac{1}{1+z}\right)^4 \frac{1}{1+z} = (1+z)^{-5} \qquad (5)$$

$$\frac{S(\lambda)}{S_{10}(\lambda)} = \left(\frac{d_{10}}{d_L}\right)^2 \cdot (1+z)^{-5} \qquad (6)$$

The fraction of the monochromatic fluxes $S(\lambda)/S_{10}(\lambda)$ does not depend on the wavelength! So the relation (6) is valid also for blue band fluxes.

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5)
The blue fluxes F and F_{10} have to be replaced by the corresponding magnitudes. Using Pogson's Ratio it can be obtained:

$$m_B - M_B = -2.5\log\left(\frac{F}{F_{10}}\right) = 5\log\left(\frac{d_L}{d_{10}}\right) + 12.5\log(1+z)$$

$$M_B = m_B - 5\log\left(\frac{d_L}{d_{10}}\right) + 12.5\log(1+z) \qquad (7)$$

Substituting numerical data one can obtain:

$$M_B = 20.40^{\mathrm{mag}} - 5\log\left(2754\cdot 10^5\right) - 12.5\log(1.5) = -24.00^{\mathrm{mag}}$$

$$\boxed{M_B = -24.00^{\mathrm{mag}}} \qquad (8)$$

Please note, that precision is determined by m_B !

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6)
Only the cD galaxies are so luminous, while the normal elliptical galaxies are substantially weaker. Accordingly, the galaxy is not a member of the cluster, but is a foreground object.

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[2] *Blue band : Effective Wavelength Midpoint λeff=445nm, FWHM=94nm [398nm÷492nm]*

Step		weigth	points
1)		10%	3
2)		30%	9
3)		10%	3
4)		30%	9
5)		10%	3
6)		10%	3
τ		100%	30