Short_long N°2

Brightness of an elliptical galaxy - text

Within the field of a galaxy cluster at a redshift of z = 0.500, an galaxy looking as normal elliptical with the apparent magnitude in the B filter m_B = 20.40 is observed.

- What is the absolute magnitude of this galaxy in the B filter?
- Can it belong to this cluster?

The luminosity distance corresponding to the redshift z = 0.500, d_L = 2754 Mpc.
A spectral energy distribution (SED) of elliptical galaxies in the range 250 nm to 500 nm is adequately approximated by a formula:

$$f(\lambda) \propto \lambda^4$$

(spectral density of flux, known also as monochromatic flux is proportional to λ^4)

Comments to liders

A spectral energy distribution (SED) is a plot of brightness or flux density versus frequency or wavelength of light. It is used in many branches of astronomy to characterize astronomical sources. For example, in radio astronomy, an SED with a negative spectral index around −0.7 would indicate a synchrotron radiation source.
The spectral index of a source is a measure of the dependence of radiative flux density on frequency. Given frequency ν and radiative flux S, the spectral index α is given implicitly by $S=C\cdot\nu^{\alpha}$. Note that if flux does not follow a power law in frequency, the spectral index itself is a function of frequency. The opposite sign convention is sometimes employed. Spectral index is also sometimes defined in terms of wavelength λ. In this case, the spectral index α is given implicitly by $S=A\cdot\lambda^{\alpha}$.
(From Wikipedia, the free encyclopedia http://en.wikipedia.org/wiki/Spectral_energy_distribution)
For distant galaxies, photometric effects of the spectrum shifts (K correction) are non-negligible. A magnitude of the K correction is related to the SED and the goal is to calculate it for filter B.
Note $m_B = M_B + DM + K_B$, DM is distance modulus defined as $5\cdot\log(d_L/10\text{pc})$.
Apparent bolometric magnitude of an object is related to its bolometric flux seen by an observer on Earth (normalized to the value it would have in the absence of the atmosphere) by the relation $m = -2.5\log(F/F_0)$, where F_0 is a reference flux (“standard” source, which for Vega-relative magnitudes is Vega). The absolute magnitude then equals to the apparent magnitude an object would have if it were at a distance equal to 10 parsecs away from the observer, in the absence of astronomical extinction. If one could measure all the light from an object at all wavelengths, a K correction would not be required. It is not a case, since only the part of the spectrum (namely B-band) is observed. Observed fraction of the total spectrum is redshifted into the frame of the observer. The measurements through a blue filter at different redshifts (d_L and 10pc) are compared so an appropriate correction is necessary.

Solution

1)
The luminosity distance is defined to satisfy a canonical inverse square law of the bolometric luminosity L (total emitted radiation power) and observed flux F (flux-luminosity relationship)

$$F = \frac{L}{4\pi d_L^2} \qquad (1)$$

Units of the flux and luminosity are expressed in SI ($W \cdot m^{-2}$ and W respectively), or CGS ($erg \cdot s^{-1} \cdot cm^{-2}$, $erg \cdot s^{-1}$ respectively).

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2)
Only the part of the spectrum (namely B-band) is observed, but the inverse square law is applicable also to the monochromatic luminosity and monochromatic flux.

Monochromatic flux is sometimes called as spectral density of flux (energy per unit time per unit area per unit wavelength) while monochromatic luminosity as spectral density of the luminosity (energy per unit time per unit wavelength). Their form is described by SED and thus spectral index.

$$S(\lambda_o)d\lambda_o = \frac{L_\lambda(\lambda_e)}{4\pi d_L^2} \cdot d\lambda_e,$$

The flux in the blue band is expressed as $F_B = \int R_B(\lambda) S(\lambda) d\lambda$, where $R_B(\lambda)$ denotes relative sensitivity in the blue bandpass.
Observed fraction of the total spectrum is redshifted into the frame of the observer, and a relationship between the observed and emitted wavelengths, λ_o and λ_{em}, can be expressed as $\lambda_{em} = \lambda_o / (1 + z)$. So,

$$\boxed{S(\lambda_o)d\lambda_o = \frac{1}{4\pi d_L^2} \cdot L_\lambda\left(\frac{\lambda_o}{1+z}\right) \cdot \frac{1}{1+z} d\lambda_o .} \qquad (2)$$

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3)
The absolute luminosity is defined as the flux would be observed at a distance of 10 pc from the compact source at rest (not redshifted):

$$S_{10}(\lambda) = \frac{L_\lambda(\lambda)}{4\pi d_{10}^2}, \qquad (3)$$

where d_{10} is the distance equal to 10 pc, S_{10} denotes a monochromatic flux in the wavelength of λ measured at 10 pc from the object.

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4)
Dividing eq. (2) by (3) we get

$$\frac{S(\lambda)}{S_{10}(\lambda)} = \left(\frac{d_{10}}{d_L}\right)^2 \cdot \frac{L_\lambda\left(\frac{\lambda}{1+z}\right)}{L_\lambda(\lambda)} \cdot \frac{1}{1+z} \qquad (4)$$

for all the wavelengths. Using the SED of the galaxy, we get

$$\frac{L_\lambda\left(\frac{\lambda}{1+z}\right)}{L_\lambda(\lambda)}\cdot\frac{1}{1+z}=\left(\frac{1}{1+z}\right)^4\frac{1}{1+z}=(1+z)^{-5}$$

$$\frac{L_\lambda\left(\frac{\lambda}{1+z}\right)}{L_\lambda(\lambda)}\cdot\frac{1}{1+z}=\left(\frac{1}{1+z}\right)^4\frac{1}{1+z}=(1+z)^{-5} \quad (5)$$

$$\frac{S(\lambda)}{S_{10}(\lambda)}=\left(\frac{d_{10}}{d_L}\right)^2\cdot(1+z)^{-5} \quad (6)$$

The fraction of the monochromatic fluxes $S(\lambda)/S_{10}(\lambda)$ does not depend on the wavelength!
So the relation (6) is valid also for blue band fluxes.

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5)
The blue fluxes F and F_{10} have to be replaced by the corresponding magnitudes.
Using Pogson's Ratio it can be obtained:

$$m_B-M_B=-2.5\log\left(\frac{F}{F_{10}}\right)=5\log\left(\frac{d_L}{d_{10}}\right)+12.5\log(1+z)$$

$$M_B=m_B-5\log\left(\frac{d_L}{d_{10}}\right)+12.5\log(1+z) \quad (7)$$

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6)
Substituting numerical data one can obtain:

$$M_B=20.40^{mag}-5\log\left(2754\cdot10^5\right)-12.5\log(1.5)=-24.00^{\mathrm{mag}}$$

$$M_B=-24.00^{\mathrm{mag}} \quad (8)$$

Please note, that precision is determined by m_B !

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7)
Only the cD galaxies are so luminous, while the normal elliptical galaxies are substantially weaker. Accordingly, the galaxy is not a member of the cluster, but is a foreground object.

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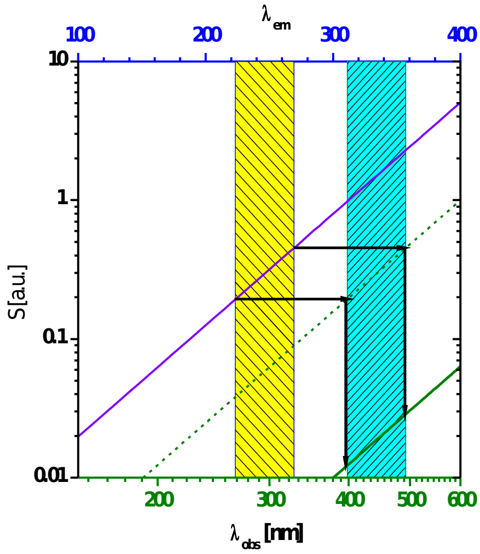
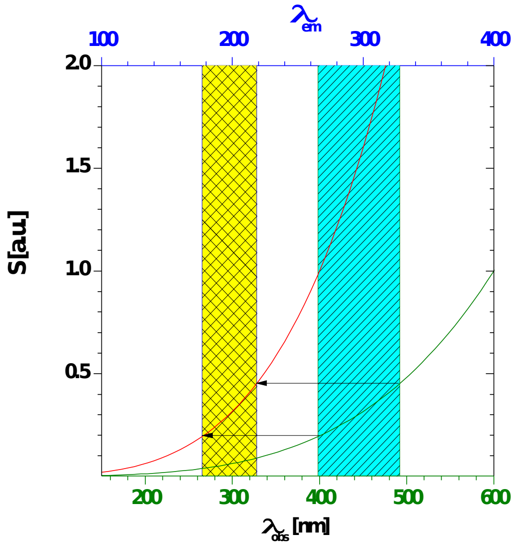
As far as calculus is concern, critical point for solution is eq. (2). But this equation can be obtained looking on the following figures





Wavelength axis is scaled by a factor (1+z), it means that band width is also transformed.

In semilog scale decreasing of the monochromatic flux is divided into horizontal shift due to the redshift and vertical due to the distance.

$$S(\lambda_o)\Delta\lambda_o = \frac{1}{4\pi d_L^2} \cdot L_\lambda\left(\frac{\lambda_o}{1+z}\right) \cdot \frac{1}{1+z}\Delta\lambda_o \, .$$