**Short theoretical questions**

1. Most single-appearance comets enter the inner Solar System directly from the Oort Cloud. Estimate how long it takes a comet to make this journey.

   Assume that in the Oort Cloud, 35 000 AU from the Sun, the comet was at aphelion.

2. Estimate the number of stars in a globular cluster of diameter 40 pc, if the escape velocity at the edge of the cluster is 6 km $s^{-1}$ and most of the stars are similar to the Sun.

3. On 9 March 2011 the Voyager probe was 116 406 AU from the Sun and moving at 17 062 km $s^{-1}$. Determine the type of orbit the probe is on: (a) elliptical, (b) parabolic, or (c) hyperbolic. What is the brightness of the Sun as seen from Voyager?

4. Assuming that Phobos moves around Mars on a perfectly circular orbit in the equatorial plane of the planet, give the length of time Phobos is above the horizon for a point on the Martian equator.

   Use the following data:

   Radius of Mars $R_{\mathrm{Mars}} = 3\ 393$ km
   Mass of Mars $M_{\mathrm{Mars}} = 6.421 \times 10^{23}$ kg
   Rotational period of Mars $T_{\mathrm{Mars}} = 24.623$ h
   Orbital radius of Phobos $R_{\mathrm{P}} = 9\ 380$ km

5. What would be the diameter of a radiotelescope working at a wavelngth of $\lambda = 1$ cm with the same resolution as an optical telescope of diameter $D = 10$ cm?

6. Tidal forces result in a torque on the Earth. Assuming that, during the last several hundred milion years, both this torque and the length of the sidereal year were constant and had values of $6 \times 10^{16}$ N m and $3.15 \times 10^{7}$ s respectively, calculate how many days there were in a year 600 million years ago.

   Assume that the Earth is a homogenous sphere of density 5 500 kg $m^{-3}$.

7. A satellite orbits the Earth on a circular orbit. The initial momentum of the satellite is given by the vector **p**. At a certain time, an explosive charge is set off which gives the satellite an additional momentum $\Delta\mathbf{p}$, equal in magnitude to $|\mathbf{p}|$.
   Let $\alpha$ be the angle between the vectors **p** and $\Delta\mathbf{p}$, and $\beta$ between the radius vector of the satellite and the vector $\Delta\mathbf{p}$.

   What should be the direction of the additional momentum vector $\Delta\mathbf{p}$, for the new orbit to become:

   (a) a hyperbola with perigee at the location of the explosion,
   (b) a parabola with perigee at the location of the explosion,
   (c) an ellipse with perigee at the location of the explosion,
   (d) a circle,
   (e) an ellipse with apogee at the location of the explosion?

   For each orbit above, if it is possible mark **YES** on the answer sheet and give values of $\alpha$ and $\beta$ for which it is possible. If the orbit is not possible, mark **NO**.

   Note that for $\alpha = 180°$ and $\beta = 90°$ the new orbit will be a line along which the satellite will free fall vertically towards the centre of the Earth.

8. Assuming that dust grains are black bodies, determine the diameter of a spherical dust grain which can remain in the vicinity of the Earth in equilibrium between the radiation pressure and gravitational attraction of the Sun.

   Take the density of the dust grain to be $\varrho = 10^3$ kg m$^{-3}$.

9. Interstellar distances are large compared to the sizes of stars. Thus, stellar clusters and galaxies which do not contain diffuse matter essentially do not obscure objects behind them. Estimate what proportion of the sky is obscured by stars when we look in the direction of a galaxy of surface brightness $\mu = 18.0$ mag arcsec$^{-2}$.

   Assume that the galaxy consists of stars similar to the Sun.

10. Estimate the energy an elementary particle would need to penetrate the Earth's magnetosphere. Assume that the penetration is perpendicular to a belt of constant magnetic field 30 μT and thickness $10^4$ km.

    (Note that at such high energies the momentum can be replaced by the expression $E/c$.)

11. Based on the spectrum of a galaxy with redshift $z = 6.03$ it was determined that the average age of the stars in the galaxy is (580 ± 20) million years. At what $z$ did the epoch of star formation occur in this galaxy?

    Assume that the current age of the Universe is $t_0 = 13.7 \times 10^9$ years and that the rate of expansion of the Universe is given by a flat cosmological model with cosmologica constant $\Lambda = 0$. (In such a model the scale factor $R \sim t^{2/3}$, where $t$ is the time since the Big Bang.)

12. Due to the precession of the Earth's axis, the region of sky visible from a location with fixed geographical coordinates changes with time. Is it possible that, at some point in time, Sirius will not rise as seen from Krakow, while Canopus will rise and set?

    Assume that the Earth's axis traces out a cone of angle 47°. Krakow is at latitude 50.1° N; the current equatorial coordinates (right ascension and declination) of these stars are:

    | | | |
    |---|---|---|
    | Sirius (α CMa) : | $6^h\,45^m$ | $-16°\,43'$ |
    | Canopus (α Car) : | $6^h\,24^m$ | $-52°\,42'$ |

13. The equation of the ecliptic in equatorial coordinates ($\alpha$, $\delta$) has the form:

    $$\delta = \operatorname{atan}(\sin\alpha \tan\varepsilon),$$

    where $\varepsilon$ is the angle of the celestial equator to the ecliptic plane.
    Find an analogous relation $h = f(A)$ for the galactic equator in horizontal coordinates ($A$, $h$) for an observer at latitude $\varphi = 49°\,34'$ at local siderial time $\theta = 0^h\,51^m$.

14. Estimate the number of solar neutrinos which should pass through a 1 m$^2$ area of the Earth's surface perpendicular to the Sun every second. Use the fact that each fusion reaction in the Sun produces 26.8 MeV of energy and 2 neutrinos.

15. Given that the cosmic background radiation has the spectrum of a black body throughout the evolution of the Universe, determine how its temperature changes with redshift $z$.
    In particular, give the temperature of the background radiation for the furthest currently observed objects with $z \sim 10$.

**Short theoretical questions – reserve**

16. Give the focal length of the eyepiece needed to best see the faintest nebulae through a telescope with an objective lens of diameter 8 cm and focal length 40 cm.

17. Due to the intrinsic variability of the Sun, the Solar irradiance varies from 1362 $Wm^{-2}$ to 1368 $Wm^{-2}$, and the effective temperature by up to 1.5 K. What is the maximum change of the Solar radius which could accompany these variations?

18. Consider a planetoid of spherical mass distribution and average density $\varrho$ rotating abut its axis with a period less than

$$T = \sqrt{(3\pi/G\,\varrho)}$$

Determine the lowest planetoidographic latitude which would allow landing on this body.

19. Observations of a satellite show that it travels from the zenith to an altitude of $h = 40^{\circ}$ in time $\Delta t = 105$ s. Determine the orbital period $T$ and radius of the circular orbit $R$ for the satellite.

Ignore the rotation of the Earth. Note that the speed of a satellite in a circular orbit of radius equal to the radius of the Earth (also called the 'first space velocity') is $\upsilon_I = 7.91$ km $s^{-1}$.

20. For any date in the Gregorian calendar, the Julian day number (JD) can be determined by the algorithm:

A = Int ( ( M + 9 ) / 12 ) + Y + 4716

B = Int ( 275 M / 9 ) – Int ( 7 A / 4 ) + 367 Y + D +1 729 317.5

C = Int ( ( A + 83 ) / 100)

JD = B – Int ( 3 ( C +1 ) / 4 )

(where: Y – year, M – month, D – day, Int ( ) – integer part)

Determine the day of the week and the date (day, month, year) of birth of Jan Hevelius, if we know that:

– it was the Julian Day which began on: JD = 2 309 492.5
– 28 August 2011 roku is a Sunday according to the Gregorian calendar.