V1

Short theoretical questions – solutions

Each question max 10 points

Question 1

Perihelium distance is << aphelium distance.

Thus, semimajor axis of the comet orbit $a \approx 17500$ AU. 1

Application of the III Kepler's Law to get the comet orbital period 4

Actual calculations: $T = a^{2/3} \approx 2.32 \cdot 10^6$ (T in years) 4

The sought solution is equal to one half of T, i.e. approx. $1.16 \cdot 10^6$ years 1

Question 2

$v_{\mathbf{c}} = \sqrt{(2GM_{sol}\, n/R)}$, 5

solve for n. (~ 100 000). 5

Question 3

Sum of kinetic and potential energy of Voyager is $E = -G\, m\, M_{\odot} / r + mv^2 / 2$ 3

Where m – mass of Voyager, r = 116.406 AU, v = 17.062 km/s.

Substituting numerical values into energy equation, 3

we get $E > 0$, therefore the orbit is hyperbolic 1

Brightness of the Sun is $m(r) = m_{\odot} + 5 \log(r)$ 2

≈ -16.5 . 1

Question 4

The orbit of Phobos and the rotation of the planet are in the same direction, so the angular velocity of the moon relative to Mars, $\omega_{P/M}$, is equal to the difference

$\omega_{P/M} = \omega_M - \omega_P$ 2

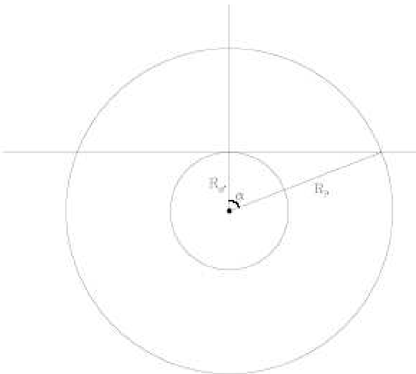
where ω_M and ω_P are the angular rotation velocity of Mars and the angular orbital velocity of Phobos respectively.

For ω_M we have $\omega_M = 2\pi / T_M$ where T_M is the period 1

The time above the horizon, t, is found from the relation (see drawing):

$t = 2\alpha / \omega_{P/M}$ 2

where $\alpha = R_M/R_P$. 1



Find the angular velocity of Phobos ω_P assuming gravity is equal to the centripetal force:

$$\omega_P = \mathrm{sqrt}(\ GM_M / R^{3/2}_P) \qquad 2$$

Substituting:

$\omega_M = 7.0882 \cdot 10^{-5}\ s^{-1}$,

$\omega_P = 2.2784 \cdot 10^{-4}\ s^{-1}$,

$\omega_{P/M} = -1.5696 \cdot 10^{-4}\ s^{-1}$

where "–" means that the moon "leads" the rotation of the planet; substitute the absolute value into the equation,

thus

$\alpha = 68^{\circ}.794 = 1.2007$ rad.

and $t \approx 15300$ s $= 4h\ 15m$. 2

Question 5

Angular resolution: sin(λ) = 1.22 λ /D 2

D_{radio}[m] =($\lambda_{radio}\cdot D_{optic}$)/ λ_{optic} = (1cm· 10cm)/550nm = (0.01 m · 0.1 m) / 0.00000055 m= 1820 m

equation of angular resolution as a function of λ and D 2

proportion of $\lambda_{radio}/D_{radio} = \lambda_{optic}/D_{optic}$ 2

proper calculation 2

and result assuming λ_{optic} in proper range: (500 – 600 nm) 2

(400 – 900 nm)

out of range

Question 6

Calculation of the moment of inertia of Earth 2

The moment of inertia is $I = \frac{2}{5} m_E R_E^2$

Equation of motion for this case $K = I \cdot \varepsilon$ 2

where ε is angular acceleration and *K* is moment of force.

Calculation of angular acceleration 1

Acceleration is equal to ε=*K*/*I*. From the assumption, the acceleration ε is constant and we find its numerical value equal to $6\cdot10^{-22}$ s^{-2}.

Calculation of the change of ε during assumed period of time 1

Multiplying this value by $1.9\cdot10^{16}$ s [600 My] we obtain $\Delta\omega=1.14\cdot10^{-5}$ s^{-1}.

Calculation of the angular frequency in the past 2

Using the definition of the angular frequency one can calculate its recent value $2\pi/86164$ s^{-1} = $7.29\cdot10^{-5}$ s^{-1}. Subtracting from it calculated above Δω one gets $\omega_{past}=6.15\cdot10^{-5}$ s^{-1}

Calculation of the day length in the past and number of days in the ancient year 2

Having angular frequency one can get ancient period of Earth rotation . Dividing sidereal year length by the obtained period one gets number of days in the ancient year equal to about 420 days.

Question 7

Limiting ourself to the orbits with peri and apogeum in the explosion point, the angle β should be equal to 90º or 270º, while the angle α should be:

1). $\alpha \in <$ 0º ; 90º) or α∈(270º ; 360º >,
2). α = 90º or α = 270º,
3). $\alpha \in$ (90º ; 120º) or $\alpha \in$(240º ; 270º),
4). α = 120º or α = 240º,
5). $\alpha \in$(120º ; 180º) or α∈(180º ; 240º),

respectively.

Taking into account other kind of orbits (without peri and apogeum) one has to make the angle β free (unfixed).

The assessment is that 2 point for each element

Question 8

Gravitational force can be expressed in the form: $F_g = \frac{4}{3}\pi \cdot r^3 \cdot \rho \cdot G \cdot \frac{M}{R^2}$ 2

The force of radiation reads $F_r = \pi \cdot r^2 \cdot C_0 / c$ 2
where r, ρ, R, G, M, C_0 are a radius and density of dust, distance form the Sun, gravitational constant, Solar mass and Solar constant respectively.
Comparing these forces one obtains $F_g = F_r$ 2
and finally one gets the dust grain radius $r = \frac{3}{4} \frac{C_0 \cdot R^2}{c \cdot \rho \cdot G \cdot M}$ 2

Substituting numerical values one obtains $r \approx 5 \cdot 10^{-7}$ m 2

Question 9

Finding that surface brightness is independent of distance 2

Relationship between m and the Sun surface brightness:
$m - \mu_\odot = -2.5 \log ((\pi \cdot \theta^2 / 4) / 1 \text{ sq. arcsec})$. 2

Calculations of $\mu_\odot = -10.79$ 1

Galaxy vs. Sun surface brightness (in magnitudes)
$\mu - \mu_\odot = 18.0 + 10.79 = 28.79$ 1

Relationship between the sought sky fraction covered by stars, x,
and the surface brightness ratio: $\mu - \mu_\odot = -2.5 \log x$ 3

Calculations $x = 10^{-0.4(\mu - \mu_\odot)} \approx 3.1 \cdot 10^{-12}$ 1

Question 10

Formulation of a penetration conditions 2

From the conditions specified in the exercise text the particle should be pass in the belt of magnetic field around quarter of the circle. If this particle leaves the belt the particle path should be a little longer. It means that the radius of this circle should be larger then the belt thickness.

Determination of the forces (centrifugal and magnetic) acting upon the particle, writing of the appropriate formulae 4

Comparison of the forces and calculations 3

Comparing centrifugal and magnetic force one obtains

$e \cdot v \cdot B = M \cdot v^2 / r$

So $e \cdot r \cdot B = p$, where *p* denotes the momentum. From the assumed approximation $e \cdot r \cdot B = E / c$

Thus definitively particle energy is described by $E = c \cdot e \cdot r \cdot B$.

Substituting numerical values one obtains $E \approx 3 \cdot 10^{-8}$ J it means $2 \cdot 10^2$ GeV

Answer and conclusion: 1

Question 11

Within assumed cosmological model the following relation is valid:

$(1+z) = \frac{R_0}{R(z)} = \left(\frac{t_0}{t(z)}\right)^{2/3}$, where R_0 and R(z) denote the current value of the scale factor and that for the redshift equal to z respectively, while t_0 and t(z) stands for the time that passed from the Big Bang. 5

For z=6.03 the appropriate time $t(z) = 7.35\ 10^8$ years. Subtracting from *t(z)* the interval equal to 560 My and 600 My one obtains $t_1 = 1.75 \cdot 10^8$ y and $t_2 = 1.35 \cdot 10^8$ y 3

what is equivalent to $z_1 = 17.3$ and $z_2 = 20.8$. 2

Question 12

At Northern Hemisphere declination of the non-rising stars $\delta < \varphi - 90^o$, where φ stands for latitude. So in Krakow $\delta < -39.9^o$. 2

Within assumed precession model declination of the star can differ from ecliptic latitude β not more than $\varepsilon = \pm 23.5^o$. So the ecliptic latitude β of the stars at the study should be determined. 2

Solving spherical triangle

$\sin\beta = \cos\varepsilon\ \sin\delta\ - \sin\varepsilon \cos\delta \cdot \sin\alpha$ 3

Substituting by numerical values for the stars at the study one obtains:

Sirius $\beta = -39^o.7$

Canopus $\beta = -75^o.9$ 1

Due to the precession, declination of Sirius can reach -63.2° so it will become non rising star.

As far as Canopus is concern its declination never exceed -52.4°, so it will never be visible. 2

Question 13

From the attached table it can be read that northern galactic pole has the following coordinates:

$$\alpha_G = 12^h 51^m;\ \ \delta_G = +27^o 08'.$$

It means that at the sidereal time $\theta = 0^h\ 51^m$ this pole is in lower culmination at the northern side and below the horizon. 2

The angle between galactic equatorial plane and the horizon is equal to

$90^o - \delta_G + 90^o - \varphi = 103^o 18'$ and this angle is an analogue of ε in the equation of the ecliptic.

Analogue of R.A. in the equation of the ecliptic is $90^o - A$ because *A* is clockwise while R.A. is counterclockwise and at the intersection of the horizon plane and galactic equatorial azimuth is equal to 90°. 2

So we get $\mathrm{tg}(h) = \sin(90^{\circ} - A)\,\mathrm{tg}(180^{\circ} - \varphi - \delta_G)$. So,
$h = \mathrm{arc\,tg}\,(\cos A\,\mathrm{tg}\,(-\varphi - \delta_G)) = \mathrm{arc\,tg}\,(-\cos A\,\mathrm{tg}\,76^{\circ}42')$. 4

Checking:
$A = 0$, $h = -76^{\circ}42'$,
$A = 90$, $h = 0^{\circ}$. 2

Question 14
Solar constant (from table) is 1366 W m^{-2}
Each reaction gives 26.8 MeV = $26.8 \cdot 1.602 \cdot 10^{-13}$ J = $4.3 \cdot 10^{-12}$ J 4
Thus there must be $1366 / 4.3 \cdot 10^{-12} = 3.2 \cdot 10^{14}$ reactions per second to produce this energy flux
4
Each reaction produces 2 neutrinos, so there must be $6.4 \cdot 10^{14}$ neutrinos $m^{-2}\,s^{-1}$. 2

Question 15
Correct choice of the wavelength: 2

If the nature of the spectrum does not change, it is sufficient to perform the calculation for one wavelength. The simplest choice is for the wavelength of the maximum (Wien's law)

$\lambda = b/T$

Understanding definition of redshift 2

From this
$z = (\lambda_a - \lambda_e)/\lambda_e$
where λ_a is the wavelength received at the Earth and λ_e is the emitted wavelength
thus $\lambda_e = \lambda_a/(z+1)$

Substituting into Wien's law, we find

$T = b \cdot (z+1)/\lambda_a$ 4

Substituting the current value of the temperature we find the result 2