PROBLEM No. 3

The table lists the astrometric positions of the star designed by S2 which is orbiting around the supermassive black hole in the center of the Milky Way, Sgr A*. The data were collected by ESO's NTT and VLT telescopes between 1992 and 2009, and published by Gillessen et al (*ApJ Letters* **707** L114 (2009)).

The table contains seven columns:
(1) row number
(2) year of the astrometric observation (AEpoch)
(3) offset in Right Ascension (oRA) relative to Sgr A*
(4) uncertainty in oRA (e_oRA)
(5) offset in Declination relative to Sgr A* (oDE)
(6) uncertainty in oDE (e_oDE)
(7) telescope used in astrometric observation (ATel)

The position of the black hole is $(0, 0)$, the offsets of S2 positions are measured in milliarcseconds (mas).

(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	1992.224	-6.4	4.6	172.0	4.7	NTT
2	1994.314	-28.5	4.8	179.0	3.4	NTT
3	1995.534	-37.3	3.8	172.1	4.3	NTT
4	1996.253	-43.4	3.6	164.4	3.6	NTT
5	1996.427	-45.9	1.9	161.5	5.3	NTT
6	1997.544	-59.0	3.4	130.4	2.8	NTT
7	1998.373	-65.3	4.7	122.1	3.5	NTT
8	1999.465	-67.5	4.4	106.0	4.1	NTT
9	2000.472	-55.3	5.0	63.9	3.1	NTT
10	2001.502	-49.3	3.8	23.8	2.1	NTT
11	2002.250	-3.1	4.0	-6.6	4.0	VLT
12	2002.335	6.6	2.7	-7.6	2.7	VLT
13	2002.393	16.3	3.8	0.0	3.8	VLT
14	2002.409	18.2	3.3	2.1	3.3	VLT
15	2002.412	17.3	3.3	2.3	3.3	VLT
16	2002.414	17.4	3.3	3.2	3.3	VLT
17	2002.488	27.8	11.5	14.9	10.4	NTT
18	2002.578	30.8	3.3	20.7	3.3	VLT
19	2002.660	33.7	3.2	27.3	3.2	VLT
20	2002.660	34.1	3.2	26.9	3.2	VLT
21	2003.214	41.1	0.3	66.6	0.4	VLT
22	2003.351	41.4	0.3	75.0	0.3	VLT
23	2003.356	40.7	0.4	74.8	0.4	VLT
24	2003.446	40.6	0.5	79.8	0.5	VLT
25	2003.451	41.3	0.4	80.4	0.4	VLT
26	2003.452	41.5	0.3	80.5	0.3	VLT
27	2003.454	40.9	0.3	80.6	0.3	VLT
28	2003.454	41.3	0.4	81.8	0.4	VLT
29	2003.550	40.9	0.3	85.3	0.3	VLT
30	2003.676	40.6	0.3	91.8	0.3	VLT
31	2003.678	41.1	0.6	91.6	0.6	VLT
32	2003.761	40.2	0.4	96.6	0.4	VLT
33	2004.240	37.2	0.9	113.2	0.9	VLT
34	2004.325	36.9	0.3	116.2	0.3	VLT
35	2004.347	36.1	0.3	117.6	0.3	VLT
36	2004.443	36.0	0.3	120.4	0.3	VLT
37	2004.513	35.3	0.3	123.1	0.3	VLT
38	2004.516	35.2	0.6	123.1	0.6	VLT
39	2004.574	34.3	0.6	123.9	0.6	VLT
40	2004.574	34.4	0.4	124.8	0.4	VLT
41	2004.664	33.6	0.3	127.2	0.3	VLT
42	2004.730	34.0	0.7	128.9	0.7	VLT
43	2005.270	28.1	0.3	143.0	0.3	VLT
44	2005.366	27.0	0.3	145.2	0.3	VLT
45	2005.467	26.3	0.4	146.9	0.4	VLT
46	2005.576	24.9	0.4	149.4	0.4	VLT
47	2006.324	17.5	0.8	161.7	0.6	VLT
48	2007.545	2.8	0.9	175.7	0.7	VLT
49	2007.550	4.1	0.4	175.2	0.4	VLT
50	2007.686	2.5	0.5	176.0	0.5	VLT
51	2007.687	1.9	0.6	176.0	0.6	VLT
52	2008.148	-4.6	0.4	179.0	0.4	VLT
53	2008.197	-5.2	0.3	179.0	0.3	VLT
54	2008.268	-6.1	0.3	180.0	0.3	VLT
55	2008.456	-8.4	0.3	180.2	0.3	VLT
56	2008.472	-8.1	0.4	180.7	0.4	VLT
57	2008.601	-10.6	0.3	180.3	0.3	VLT
58	2008.708	-11.4	0.3	181.2	0.3	VLT
59	2009.185	-17.3	0.7	181.1	0.7	VLT
60	2009.273	-18.0	0.3	181.2	0.3	VLT
61	2009.300	-18.5	0.3	181.3	0.3	VLT
62	2009.303	-18.2	0.3	181.5	0.3	VLT
63	2009.336	-18.3	0.3	181.2	0.3	VLT
64	2009.336	-18.4	0.4	181.2	0.4	VLT
65	2009.371	-18.6	0.3	181.0	0.3	VLT
66	2009.505	-20.1	0.3	181.2	0.3	VLT
67	2009.557	-20.2	0.3	181.4	0.3	VLT
68	2009.557	-20.9	0.4	181.5	0.4	VLT
69	2009.606	-21.2	0.3	181.5	0.3	VLT

a) On a graph paper plot as many (oRA, oDE) positions of S2 as you can. If you use the scale factor of $1\,\mathrm{mas\,mm^{-1}}$ and scale the entire oRA axis between -100 and 80, and the entire oDE axis between -50 and 220, then you can plot the points quite easily. After correct plotting of the data the elliptical orbit of S2 around Sgr A* will be outlined. (15 p)

b) While plotting try to estimate the orbital period of S2 in years. (10 p)

Hint: The data given in the table cover a slightly longer time span than the orbital period.

c) Estimate the semi-major axis of the orbit in milliarcseconds. (10 p)

Hints: From the accurate orbit fitting by Gillessen et al (2009), the eccentricity is $e \approx 0.88$, while the inclination of the orbit is $i \approx 135°$. S2 last approached the black hole on 19 May 2018. Remember the value of the orbital period determined in the previous task.

d) According to Gillessen et al (2009) the distance to the center of the Milky Way is $R_0 \approx 8.3\,\mathrm{kpc}$. Give the semi-major axis of the orbit in Astronomical Units (AU). (10 p)

e) Calculate the mass of the supermassive black hole Sgr A*. (15 p)

(60 p)

SOLUTION:

a) See the plot on a separate page at the end of the solution. The long curved arrow shows the orbiting direction. The #1 the #54 data points are overplotted by a red diamond, see the point b) for the explanation. (15 p)

b) While plotting one should recognize that the orbital curve turns back almost to its starting point #1 around the point #54. So the orbital period equals to the difference of the the epochs in table rows #54 and #1:

$$P \approx 2008.268 - 1992.224 = 16.04\,\mathrm{yr} \qquad (10\,\mathrm{p})$$

In fact, plotting the data is only neccessary to recognize this step.

c) Knowing the orbital period, from the date of the last approach (19 May 2018 $= 2018.381 = t_i$), one can determine the approximate date of the previous close approach:

$$t_{i-1} = t_i - P = 2018.381 - 16.04 = 2002.337 \approx 2002.335 \qquad (2\,\mathrm{p})$$

The last number is the date in the table row #12, so we can say that the previous approach happened at 2002.335.

The offsets in R.A. and in Dec. in mas at that time:

$$\Delta\alpha_{2002} = 6.6,\ \Delta\delta_{2002} = -7.6 \qquad (2\,\mathrm{p})$$

Because of the small angles the projected pericenter distance (what we observe) in mas can be calculated as:

$$\Delta = \sqrt{\Delta\alpha^2 + \Delta\delta^2} = \sqrt{(6.6)^2 + (-7.6)^2} = 10.1 \qquad (2\,\mathrm{p})$$

As the inclination of the orbit is about $i \approx 135°$, the real pericenter distance in mas is about 1.41 times larger:

$$r_\mathrm{p} = 1.41 \times \Delta = 1.41 \times 10.1 = 14.2 \qquad (2\,\mathrm{p})$$

Now the semi-major axis in mas can be calculated as:

$$a_\mathrm{mas} = \frac{r_\mathrm{p}}{1-e} = \frac{14.2}{1-0.88} = 119 \qquad (2\,\mathrm{p})$$

d) From the distance $R_0 \approx 8300\,\mathrm{pc}$ this angular size in mas equals to a physical distance in AU of

$$a_{\mathrm{AU}} = \frac{a_{\mathrm{mas}}}{3600} \times \frac{\pi}{180} \times R_0 \times 206\,265$$

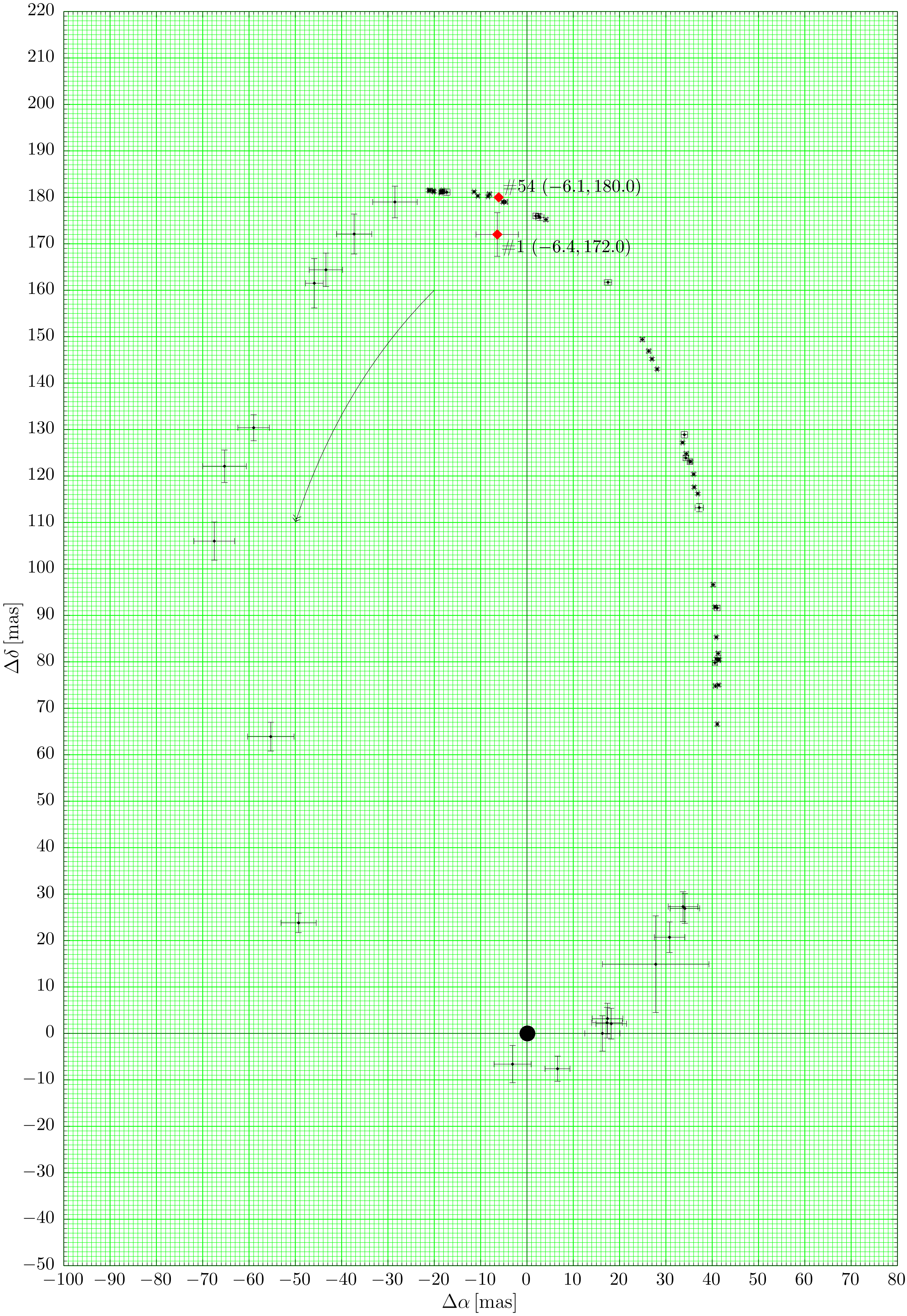
$$a_{\mathrm{AU}} = \frac{119}{3600} \times \frac{\pi}{180} \times 8300\,\mathrm{pc} \times 206\,265 = 985\,\mathrm{AU} \qquad (10\,\mathrm{p})$$

e) Because $M_{\mathrm{Sqr\ A*}} \gg M_{\mathrm{S2}}$, from the Kepler's third law

$$\frac{a^3}{P^2} = M_{\mathrm{Sqr\ A*}}, \qquad (5\,\mathrm{p})$$

where a is given in Astronomical Units, P is given in years, and M is given in solar masses.

$$M_{\mathrm{Sqr\ A*}} = \frac{985^3}{16.04^2} = 3.7 \times 10^6\,\mathfrak{M}_\odot \qquad (10\,\mathrm{p})$$



#54 (−6.1, 180.0)
#1 (−6.4, 172.0)
Δδ [mas]
Δα [mas]