IWAA 2018 - Zánka, Hungary
The Cosmic Distance Ladder - Solutions

Gergely Dálya, dalyag@caesar.elte.hu

September 3, 2018

Problem 1 - Life of an ancient astronomer

Let us denote the Sun as S, the Moon as M and the Earth as E from now on. Since we observe at a lunar dichotomy, the SME angle is 90°. We measure, that the MES angle is 87°, from which:

$$\frac{\mathrm{EM}}{\mathrm{ES}} = \cos 87° = 0.0523 \tag{1}$$

So the Sun should be **19 times** as far as the Moon. Since their apparent sizes are nearly the same, the Sun's real size should be 19 times bigger than the Moon's real size as well.

By knowing the real distances of these two celestial bodies ($d_\mathrm{S} = 149.6 \cdot 10^6$ km, $d_\mathrm{M} = 384\,400$ km), we can calculate the real MES angle:

$$\arccos \frac{d_\mathrm{M}}{d_\mathrm{S}} = 89.85° \tag{2}$$

So the real angle is **89.95°**.

Problem 2 - Moon radar

Radio waves propagate with the speed of light, so the time needed to travel back and forth to the Moon:

$$t = 2\,\frac{d_\mathrm{M}}{c} = \mathbf{2.56\ s} \tag{3}$$

The solution for the amplification problem is the following. Let us assume, that you receive a continuous signal which consists of the real signal (evenly separated flashes) and a random noise. If you know the exact period at which you emit the signals, you can cut out just those parts of the data which contain the real signal, and add them together. By this method, the signal will increase, as well as the noise, but since the latter is random, the signal-to-noise ratio will increase.

Problem 3 - Exoplanet

The period of the planet is $T = 67$ days and the mass of the star is $M = 0.31\ M_\odot = 6.2 \cdot 10^{29}$ kg. Using Kepler's third law:

$$\frac{a^3}{T^2} = \frac{GM}{4\pi^2} \quad \rightarrow \quad a = \sqrt[3]{T^2 \frac{GM}{4\pi^2}} = 3.274 \cdot 10^{11}\ \mathrm{m} = \mathbf{0.218\ AU} \tag{4}$$

Problem 4 - Stellar positions

The position of the star changed with $7.436 \cdot 10^{-6}$ radians in half a year. Based on this, the parallax of the star (π) will be exactly half of this value:

$$\pi = 3.718 \cdot 10^{-6} = 0.767''$$

The distance is the reciprocal of the parallax, if we measure the parallax in arcseconds and the distance in parsecs:

$$d = \frac{1}{\pi}$$

From this, the distance of the star is **1.304 pc**. The star can not be other than **Proxima Centauri**.

Problem 5 - Cepheids

We are going to calculate the distance of the Cepheid by the Pogson law, but first we need the two magnitudes for this. The pulsation period can be find out from Figure 2. I'd advise to read the time difference between the two farthest minima (or maxima) and divide it with how much cycles have elapsed between the two. This process will minimize the error of reading the values. The average apparent magnitude can be read from Figure 2 as well, let's calculate with $m = 14.5^m$.

Using Figure 1 we can read the absolute magnitude based on the period. While reading the data, beware of the x-axis: its scale is not linear but logarithmic! The absolute magnitude is approximately $M \approx -4.4^m$.

$$m - M = -5 + 5\log d \quad \rightarrow \quad d = 10^{\frac{m-M+5}{5}} \tag{5}$$

from where $\boldsymbol{d \approx}$ **60 kpc**

This is the distance of the Small Magellanic Cloud, which is really a member of the Local Group, so our result is consistent with the data given in the problem.

b) Let us take the interstellar extinction as well into consideration:

$$m - M = -5 + 5\log d + A \quad \rightarrow \quad d = 10^{\frac{m-M+5-A}{5}} \tag{6}$$

from where $\boldsymbol{d \approx}$ **54 kpc**

c) Let us read two pair of values from Figure 1, since we have 2 unknown constants. For example: $M_1 = -5^m$, $P_1 = 20^{\mathrm{d}}$ and $M_2 = -6.5^m$, $P_2 = 70^{\mathrm{d}}$. Substitute the first pair into the P-L relation:

$$-5 = \beta \log 20 + C \quad \rightarrow \quad C = -5 - \beta \log 20 \tag{7}$$

We are going to substitute this into the equation with the second pair:

$$-6.5 = \beta \log 70 + C \quad \rightarrow \quad -6.5 = \beta \log 70 - 5 - \beta \log 20 \tag{8}$$

After sorting the terms we get

$$\beta = \frac{-1.5}{\log 70 - \log 20} \simeq \mathbf{-2.76} \tag{9}$$

And by substituting this to Equation 7: $\boldsymbol{C = -1.41}$

Problem 6 - Supernovae

Let us estimate that the absolute magnitude of the supernova is the average absolute magnitude of Ia-type supernovae, so $M = -19.3^{\mathrm{m}}$. The naked eye can see objects as dim as 6^{m}. By using the Pogson formula:

$$6 + 19.3 = -5 + 5 \log d \quad \rightarrow \quad d \simeq 1.15 \text{ Mpc.} \tag{10}$$

So if there were no interstellar extinction, we could see Ia-type supernovae with the naked eye from as far as **1.15 Mpc**.

Super-Chandrasekhar mass supernovae can be created by the double degenerate scenario. The two progenitor white dwarfs' masses can not be higher than the Chandrasekhar mass (or they would blow up before merging), so the theoretical limit is two times the Chandrasekhar mass, i.e. **2.88 solar masses**.

Problem 7 - PNLF

As a first step we should write the data from the text of the problem into the $N(m) \sim e^{0.307m}(1 - e^{3(m^*-m)}$ equation. We will get two equations, which we should solve (k is a constant):

$$3 = k \cdot e^{0.307m_1}(1 - e^{3(m^*-m_1)}) \tag{11}$$

$$5 = k \cdot e^{0.307m_2}(1 - e^{3(m^*-m_2)}) \tag{12}$$

Let us divide the equations with each other. This way the constant k will disappear and we will have only m^* in our equation:

$$\frac{3}{5} = e^{0.307(m_1-m_2)} \cdot \frac{1 - e^{3m^*} \cdot e^{-3m_1}}{1 - e^{3m^*} \cdot e^{-3m_2}} \tag{13}$$

Rearranging a bit:

$$\frac{3}{5} - e^{0.307(m_1-m_2)} = (\frac{3}{5}e^{-3m_2} - e^{0.307(m_1-m_2)}e^{-3m_1})e^{3m^*}. \tag{14}$$

From this form we can get m^*:

$$m^* = \frac{1}{3} \ln \frac{\frac{3}{5} - e^{0.307(m_1-m_2)}}{e^{-3m_2}\left(\frac{3}{5} - e^{-2.693(m_1-m_2)}\right)} \tag{15}$$

By substituting the given m_1 and m_2:

$$\boldsymbol{m^* = 22.56^m} \tag{16}$$

By putting this result into the distance modulus formula and by using the given $M^* = -4,48$ value, for the distance we get:

$$\boldsymbol{d = 2.56 \text{ Mpc}} \tag{17}$$

Problem 8 - Spiral galaxy populations

As a first step, you should calculate the rotational velocities of the galaxy sample by using Doppler's law:

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}, \tag{18}$$

where $\Delta\lambda$ is the difference between the observed and the rest-frame wavelength (21 cm), so $\Delta\lambda = \lambda_{\mathrm{max}} - 21$ cm. By finding the velocities, you can plot the magnitudes and the velocities. Note, that either plot the log of the velocities or use log scale on the velocity axis. You'll get a figure like Figure 1.

On the plot two distinct populations of galaxies clearly emerge. In fact, they are Sa and Sb type spirals. Both of the types follow the Tully-Fisher relation, but with different constants: you can fit different straight lines to the two populations, see Figure 2. After fitting the two lines, you can calculate the constants of the Tully-Fisher relations by getting the coordinates of two points in each fitted line, just as we did in Problem 5. The final forms of the two relations will be the following:

$$\textbf{Sa galaxies :}\ \boldsymbol{M_{\mathrm{B}} = -9.95 \log_{10} V_{\mathrm{max}} + 3.15} \tag{19}$$

$$\textbf{Sb galaxies :}\ \boldsymbol{M_{\mathrm{B}} = -10.2 \log_{10} V_{\mathrm{max}} + 2.71} \tag{20}$$

Problem 9 - Expanding Universe

Let us denote the Milky Way with T. From the text of the problem we know the G-T and the F-T distances as well as the GFT angle as seen on Figure 3.

The missing distance, x can be calculated by using the cosine law (of ordinary triangles, no spherical stuff here):

$$3^2 = x^2 + 2.5^2 - 2 \cdot x \cdot 2.5 \cdot \cos\varphi \tag{21}$$

which could be rearranged to the following quadratic equation:

$$x^2 - 2.083\,x - 2.75 = 0 \tag{22}$$

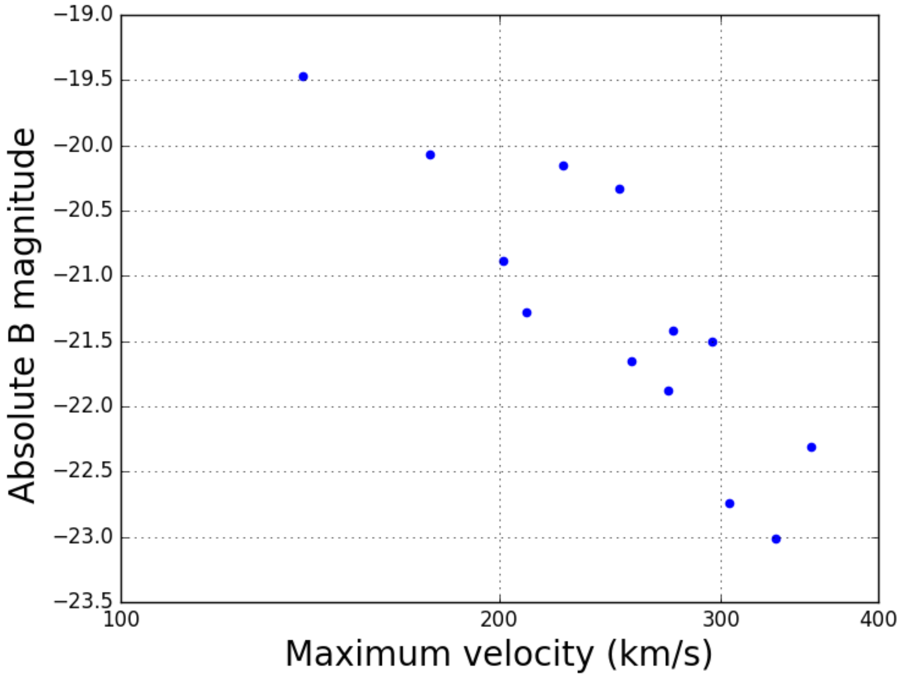

Figure 1: Rotational velocities and absolute magnitudes of the galaxy sample.

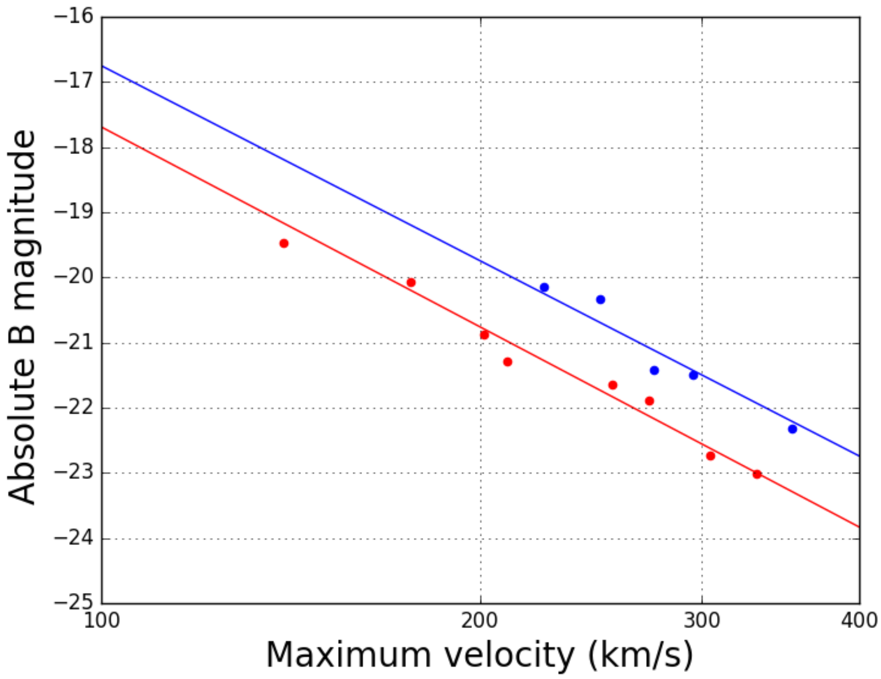

Figure 2: Rotational velocities and absolute magnitudes of the galaxy sample, with the fitted lines. Red and blue dots represent two different classes of spiral galaxies.

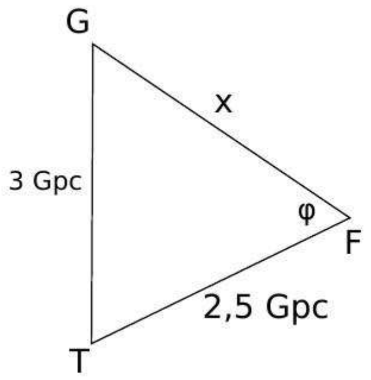

Figure 3: The location of the three galaxies

Its positive solution is 3, so F is located 3 Gpc away from G. Using Hubble's law the recession velocity of F from G could be derived:

$$Hx = v = 210\,000\ \frac{\text{km}}{\text{s}} = 0.7\ \text{c} \tag{23}$$

At such high velocities the relativistic form of Doppler's law should be used to calculate the shift in wavelength:

$$\frac{\Delta\lambda}{\lambda} = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} - 1 = 1.38 \tag{24}$$

So the Hα line, which has a rest-frame wavelength of $\lambda = 656,28$ nm will be shifted by $\Delta\lambda = 905,98$ nm, so the line will be visible at $\boldsymbol{\lambda_{obs} = 1562.26}$ **nm**.